The Role of Firm Heterogeneity in Earnings Inequality

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Abstract

Over the past three decades, individual earnings inequality has risen alongside increases in the concentration of firm employment and revenue in the U.S. This paper studies the factors underlying these trends and their macroeconomic impacts. I extend a canonical uninsurable earnings risks model with heterogeneous firms and labor market search friction as in Lucas and Prescott (1974). There are a large number of spatially distinct labor markets – islands or firms – and workers' earnings become a product of their own labor productivity and a function of employers' productivity. Workers may leave an island by paying search cost or can be exogenously separated. Once they leave, they move to a nearby island following a transition process.

Through searching, better workers move to better firms. Better workers can afford search cost since they tend to be wealthy. Also, once they move to a better firm, their wage increase is larger than the wage increase of low-productivity workers. Thus, productive workers engage in searching and have a higher chance to move up, while low-productive workers tend to stay where they are.

The model successfully replicates earnings distribution, individual and firm factors in earnings variance, and firm size distribution. With the quantitatively disciplined model, a counterfactual exercise is designed to decompose the factors affecting the rise in earnings inequality. It shows that the individual component in wages explains most of the rise in earnings inequality. Surprisingly, the changes in the firm productivity distribution and the worker allocation over firms do not contribute to the rise in earnings inequality but mitigate it.

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1. Introduction

Over the past three decades there has been a significant rise in U.S. earnings inequality. While most macroeconomic theories about earnings inequality frequently assume that individuals' earnings depend solely on their ability, Song, Price, Guvenen, Bloom, and Von Wachter (2015) show that the rise in earnings inequality among workers has primarily been a between-firm phenomenon. Using earnings data from W-2 records held by the Social Security Administration (SSA), they find that over two-thirds of the increase in earnings inequality from 1981 to 2013 can be accounted for the rising variance of earnings between firms. At the same time, we know that the firm size (in terms of employment and revenue) distribution is highly left skewed. Autor, Dorn, Katz, Patterson, and Van Reenen (2017) show that the concentration of firms is extreme and industries become increasingly dominated by a few superstar firms.[1]

These facts lead to interesting questions. How does rising firm inequality affect trends in earnings inequality? If there is a systemic relationship, what is the mechanism behind these observations? What will be the effect of this trend on the growth? This paper seeks to understand

[1] Further, they relate this trend to a falling labor share. Since superstar firms tend to have high profits and a low share of labor in their value added and sales, as the importance of these firms increases, the aggregate labor share tends to fall. Using manufacturing data, they show that industries where concentration rose the most were also those where the labor share fell by the most. Interestingly, while the labor share fell, average wages were not systematically falling.

earnings inequality and firm inequality together, find a coherent mechanism that accounts for these trends, and quantify the factors affecting the trends and their macroeconomic consequences.

To answer these questions, I construct a model with uninsurable earnings risk as in Huggett (1993) and Aiyagari (1994), but I add heterogeneous firms and search friction in the labor market. In Huggett-Aiyagari type models there is a representative firm that hires all workers. Given a lack of firm heterogeneity, all earnings differences are the result of workers' skill. While convenient, this abstracts away the rich heterogeneity in firms and, as a result, misses an important determinant of earnings differences.

The search friction follows Lucas and Prescott (1974). There is a continuum of islands, and islands differ by their productivities. In the beginning of a period, workers in each island decide whether to stay or to move to other islands. If a worker searches, she must pay search cost, and she moves to a nearby island following a job transition process.

With heterogeneity in both workers and firms, I am able to study how firms and workers affect the earnings distribution and how it has changed over time. In this environment, an earnings distribution is shaped by workers' labor productivity distribution, firm productivity distribution, and matching between the two. Since these elements presumably vary over time, it is not clear what drives the rise in the variation of workers' earnings across firms.

To see causes and consequences of rising inequalities, I perform a counterfactual exercise. I calibrate the model targeting the data circa 1992 and 2013. The model is calibrated to match the earnings distribution, individual and firm components in earnings variance, and the firm size distribution. The model captures the rise in earnings variance as well as the rise in segregation (similar workers became increasingly likely to work with each other) and positive sorting (high-skill workers became increasingly likely to work at high-wage firms), which are measured by components of variance decomposition results. Although I do not target moments of the wealth distribution, it matches the wealth distribution fairly well, except at the top 1%. Also, the model captures features of earnings dynamics that are described in Guvenen, Karahan, Ozkan, and Song (2015) reasonably well.

The changes between the two periods originate from varying labor/firm productivity processes and the changes in matching between firms and workers. The calibration identifies how these processes have shifted to induce the changes we observe in the data. Then I compute equilibria without changes in (1) the labor productivity process, (2) the firm productivity process, and (3) the job transition process, search cost, and separation rate. The quantitative exercises show that changes in the labor productivity process – the individual component – explain most of the rising earnings inequality. Without changes in the labor productivity process, the rise in inequality measured by earnings variance, the earnings share of the top 10%, and the 90/10 ratio disappears. Surprisingly, the changes in the firm productivity process and the parameters that mainly affect worker allocation over firms mitigate the rise in earnings inequality. Without changes in the search cost, separation rate, and job transition process, the employment share of large firms is smaller than in 1992, and sorting and segregation are close to the 1992 level. However, the earnings share of the top 20%, top 10%, and top 1% are larger in this hypothetical economy than in 2013 because of the changes in worker composition within firms. When shut off the changes in job transition process, low-skill workers are more likely to work at high-wage firms, and high-skill workers are more likely to work at low-wage firms (less sorting and segregation). Therefore total earnings decrease, and the earnings share of high-skill workers who work at high-wage firms – and become top earners – increases.

I keep the available resources – total effective labor and technology – constant over the periods. This allows me to assess implications of rising inequality to macroeconomic variables such as output growth. The model suggests that the rising inequality accounts for 2.0% of the output growth, 3.5% of the capital growth and 15% of decline of the interest rate since the 1990's. The output growth is driven by capital growth and increasing concentration to large, productive firms. Among other factors, positive sorting plays a role. As productive workers are increasingly matched with productive firms, total output increases. The capital growth and the decline of the interest rate are caused by changes in workers' saving decisions. From 1992 to 2013, the variances of labor productivity increase, which means higher earnings risk for workers;[2] hence, the majority of them try to increase savings. The aggregate capital demand also rises, but not as much as the rise in the supply; thus, the interest rate falls.

Firm heterogeneity also affects workers' savings behavior. Unlike models that assume a representative firm, in my model earnings risks come from various sources: labor productivity shocks, shocks to workers' employers, and separations. Therefore workers tend to save more, especially low-income households increase savings more than high-income households in response to the additional earnings risks. Therefore when one collapses firm heterogeneity, wealth concentration increases while earnings concentration decreases.

The remainder of the paper is structured as follows. Section 2 introduces motivation and literature. Section 3 lays out the model. Section 4 presents calibration results. Section 5 describes the accounting and the effect of rising inequalities to the U.S. economy. Section 6 concludes and discusses avenues for future research.

[2] However, the variance of individuals' earnings growth rates is stable over the periods.

2. Motivation and Literature

Research into the role of firm heterogeneity in the rise of earnings inequality is motivated by several reasons. A growing body of research reports that firm or establishment heterogeneity accounts for the majority of the rise in earnings variance. In addition to Song et al. (2015), Barth, Bryson, Davis, and Freeman (2014) show that the increase in earnings inequality is a result of the increased dispersion of earnings among the establishments where individuals work using Current Population Survey (CPS), Longitudinal Business Database (LBD), and Longitudinal Employer-Household Dynamics (LEHD) data. Faggio, Salvanes, and Van Reenen (2010) as well as Card, Heining, and Kline (2013) found the same pattern using U.K. and German data. Alvarez, Benguria, Engbom, and Moser (2018) show the decline of earnings inequality in Brazil, yet they that firm heterogeneity is important in explaining the decline as well. This is in contrast to other widely studied margins of inequality, such as labor demand side explanations (e.g. skill-biased technological change; see Acemoglu (2002), Katz and Murphy (1992), Goldin and Katz (1998), Autor, Katz, and Krueger (1998), and Krusell, Ohanian, Ríos-Rull, and Violante (2000)), and labor supply side explanations (see Card and Lemieux (2001)). These papers consider heterogeneity among workers and investigate what causes the changes in the price of certain types of labor. However, they abstract from firm heterogeneity.

The model provides a structural interpretation of the existing empirical research. While many papers report the rising variation of individuals' earnings across firms and the increasing positive sorting between workers and firms, it is not clear where this comes from. It might be the result of changes in individuals' skill dispersion, or changes in firm compensation differences.

Also, the two-sided heterogeneity in the model makes it possible to study the link between inequality, productivity, and growth. Banerjee and Duflo (2003) and Forbes (2000) investigate how inequality affects growth using cross-country data. In my model, the matching between workers and firms is important in shaping earnings distribution but also affects total factor productivity as well as the total output of an economy. Therefore inequality does not cause growth or vice versa, but they do affect each other. There is no definitive relationship between inequality and growth, and the model provides a laboratory to quantitatively assess the effects that changes in matching between workers and firms have on inequality and growth.

3. Model

The economy runs infinitely. There is a measure of households and a continuum of islands. On each island there is a continuum of firms. Each island has different productivity, but firms are identical within an island. Households differ by their labor productivities, assets, and islands.

3.1. Households

A household is characterized by its asset, $a \in \mathbf{B} \subset \mathbf{R}$, a labor productivity, ε, and an island, z. The labor productivity varies according to Markov processes, $\Gamma_{\varepsilon\varepsilon'}$; $\varepsilon \in \mathbf{E} \equiv \{\varepsilon_1, ..., \varepsilon_{N_\varepsilon}\}$, where $Pr(\varepsilon' = \varepsilon_j | \varepsilon = \varepsilon_i) = \pi_{ij} \geq 0$ and $\sum_{j=1}^{N_\varepsilon} \pi_{ij} = 1$ for each $i = 1, .., N_\varepsilon$. $\varepsilon_{N_\varepsilon} = 0$, which captures retirees with zero labor income.[3] Workers with labor productivity from ε_1 to $\varepsilon_{N_\varepsilon - 1}$ retire with the probability p_r; hence, $\pi_{iN_\varepsilon} = p_r$ for $i = 1, .., N_\varepsilon - 1$. After retirement, an agent dies with the probability p_d. Once dead, the agent is born with the wealth inherited from the deceased and a labor productivity that is drawn from the ergodic distribution of ε. When a household starts working, they will be allocated over islands following $\pi(z|\varepsilon)$ which will be described later. Workers' employers' productivity z also varies according to a Markov processes, $\Gamma_{zz'}$; $z \in \mathbf{Z} \equiv \{z_1, ..., z_{N_z}\}$, where $Pr(z' = z_j | z = z_i) = \pi_{ij} \geq 0$ and $\sum_{j=1}^{N_z} \pi_{ij} = 1$ for each $i = 1, .., N_z$.

I summarize the distribution of households over (ε, a, z) using probability measure μ_h defined on $\mathbf{S}$, where $\mathbf{S} = \mathbf{E} \times \mathbf{B} \times \mathbf{Z}$. Firm probability measure, μ_f, is defined over $z \in \mathbf{Z}$, and μ indicates the whole distribution

[3] In the Survey of Consumer Finance, a substantial portion of the sample has no labor income. For example, 22.5% of the sample has zero or negative earnings in 1998 (Rodriguez, Díaz-Giménez, Quadrini, and Ríos-Rull (2002)).

of the economy. The aggregate state of the economy is μ, which evolves overtime according to a mapping, Γ; $\mu' = \Gamma(\mu)$. Household distribution evolves in part by their searching decision and savings decision, and in part by shocks – ε, z shocks and unemployment shock. Firm distribution evolves solely by shock to z.

Worker A household on an island z, who is not retired or unemployed, works, earns labor income, and saves. At the end of the period, the household may choose to leave the island. If she does so, she has to pay search cost s and will be allocated to an island. The household cannot choose the destination but knows the probability of arriving over islands, $\pi(z|\varepsilon)$, which depends on workers' labor productivity. Aside from its labor productivity shocks, and shocks to its employers, a household may become unemployed with probability p_u which is equivalent to being sent to an island with zero productivity. A household solves the following problem when she decides to search:

$$
\begin{aligned}
W_s(\varepsilon, a, z; \mu) = \max_{a'} u(c) + \beta p_u \sum_{\varepsilon'} \Gamma_{\varepsilon\varepsilon'} W(\varepsilon', a', 0; \mu') \\
\beta(1 - p_u) \int_z \pi(z|\varepsilon) (\sum_{z'} \Gamma_{zz'} \sum_{\varepsilon'} \Gamma_{\varepsilon\varepsilon'} W(\varepsilon', a', z'; \mu')) dz
\end{aligned}
$$

$$
c + a' + s = (1 + (1 - \tau_c(T(w(z;\mu)\varepsilon)))r(\mu))a + w(z;\mu)\varepsilon - T(w(z;\mu)\varepsilon), \;\; a' > \underline{a}
$$

$$
\mu' = \Gamma(\mu)
$$

Workers pay income taxes according to a progressive income tax system. I assume a tax function for labor income takes the form

$$T(y) = max(min(y - \tau_i(y)^{1-\gamma}, \tau_{max}y), \tau_{min}y),$$

where γ indicates progressivity of the tax system: a larger γ means a more progressive income tax. Capital income is progressive too – the tax rate τ_c depends on the labor income tax rate.

The household may prefer stay on the island. Even if the worker decides to stay, she can be exogenously moved to another island with probability δ at the end of the period. Although the worker chooses to stay and not be separated, the firm could be hit by productivity shock, z', so the wage rate of this worker can be changed in the next period. The worker's problem is

$$\begin{aligned} W_e(\varepsilon, a, z; \mu) = \max_{a'} u(c) + \beta p_u \sum_{\varepsilon'} \Gamma_{\varepsilon\varepsilon'} W(\varepsilon', a', 0; \mu') \\ \beta(1 - p_u)[(1 - \delta) \sum_{z'} \Gamma_{zz'} \sum_{\varepsilon'} \Gamma_{\varepsilon\varepsilon'} W(\varepsilon', a', z'; \mu') + \\ \delta \int_z \pi(z|\varepsilon)(\sum_{z'} \Gamma_{zz'} \sum_{\varepsilon'} \Gamma_{\varepsilon\varepsilon'} W(\varepsilon', a', z'; \mu'))dz] \end{aligned}$$

$$c + a' = (1 + (1 - \tau_c(T(w(z; \mu)\varepsilon)))r(\mu))a + w(z; \mu)\varepsilon - T(w(z; \mu)\varepsilon), \;\; a' > \underline{a}$$

$$\mu' = \Gamma(\mu).$$

Unemployed An unemployed person moves to productive islands with probability p_{em} and becomes a worker. When back to work, she will be allocated to islands following $\pi(z|\varepsilon)$.[4] While unemployed, she receives unemployment benefit, b, from government. The unemployed problem is the following:

$$W(\varepsilon, a, 0; \mu) = \max_{a'} u(c) + \beta(1 - p_{em}) \sum_{\varepsilon'} \Gamma_{\varepsilon\varepsilon'} W(\varepsilon', a', 0; \mu')$$
$$\beta p_{em} \int_z \pi(z|\varepsilon)(\sum_{z'} \Gamma_{zz'} \sum_{\varepsilon'} \Gamma_{\varepsilon\varepsilon'} W(\varepsilon', a', z'; \mu'))dz$$

$$c + a' = (1 + (1 - \tau_c(b))r(\mu))a + b, \;\; a' > \underline{a}$$

$$\mu' = \Gamma(\mu)$$

Retirees z does not affect retirees, their income is transfer, τ_r. They die with probability p_d and they will be replaced with new-borns once dead. As in Castaneda, Diaz-Gimenez, and Rios-Rull (2003), I assume that when a retired household dies, it does so after it made the current period's consumption and savings decision. The new-borns draw ε from the ergodic distribution of ε and wealth inherited from the deceased after paying estate tax, $\tau_E(a)$. Retirees solve the following problem:

[4] Although unemployed households may search, since the searching decision does not affect the probability of becoming employed, p_{em}, no one will search.

$$W(\varepsilon, a; \mu) = \max_x u(c) + \beta \sum_{\varepsilon'} \Gamma_{\varepsilon\varepsilon'} W(\varepsilon', a'(x); \mu')$$

$$c + x = (1 + (1 - \tau_c(\tau_r))r(\mu))a + \tau_r, \;\; a' > \underline{a}$$

$$\begin{cases} a'(x) = x\,, & \textbf{if } \varepsilon' = \varepsilon_{N_\varepsilon} \\ a'(x) = x - \tau_E(x), \,, & \textbf{if } \varepsilon' < \varepsilon_{N_\varepsilon} \end{cases}$$

$$\mu' = \Gamma(\mu)$$

Workers compare two values and decide whether to search. The unemployed's value is $W(\varepsilon, a, 0; \mu)$, and the retiree's value is $W(\varepsilon, a; \mu)$.

$$\begin{cases} W(\varepsilon, a, z; \mu) = \max[\, W_e(\varepsilon, a, z; \mu), W_s(\varepsilon, a, z; \mu)\,], & \textbf{if } \varepsilon < \varepsilon_{N_\varepsilon} \textbf{ and } z > 0 \\ W(\varepsilon, a, 0; \mu), & \textbf{if } \varepsilon < \varepsilon_{N_\varepsilon} \textbf{ and } z = 0 \\ W(\varepsilon, a; \mu), & \textbf{if } \varepsilon = \varepsilon_{N_\varepsilon} \end{cases} \tag{1}$$

Let $\chi(\varepsilon, a, z)$ be households' decision rule for searching and $A(\varepsilon, a, z)$ be the decision rule for savings.

3.2. *Firms*

An island is characterized by its productivity, z, which follows a Markov process. There is a continuum of identical firms on island; thus, the labor market is competitive within an island. Firms hire workers and rent capital, k, with price $r + \delta_d$. Using labor and capital, they produce goods with CRS technology. A firm's problem in on island is

$$V(z;\mu) = \max_{\overline{\varepsilon},k} f(z,\overline{\varepsilon},k) - w(z;\mu)\overline{\varepsilon} - (r(\mu) + \delta_d)k \tag{2}$$

$$f(z,\overline{\varepsilon},k) = zk^{\alpha}\overline{\varepsilon}^{1-\alpha}$$

$$\overline{\varepsilon} = \int_{s_i \in s} \varepsilon\mu_h(\varepsilon, a, z_i)d(\varepsilon \times a \times z)$$

where i indicates an island and $\overline{\varepsilon}$ is the sum of effective labor on the island. Since the labor market is competitive on the island, wage rate is a marginal productivity of labor, $w(z,\mu) = f_{\overline{\varepsilon}}(z,\overline{\varepsilon},k) = (1-\alpha)z(\frac{k}{\overline{\varepsilon}})^{\alpha}$. I assume a frictionless capital market so firms will rent capital where $r(\mu) + \delta_d = f_k(z,\overline{\varepsilon},k) = \alpha z(\frac{k}{\overline{\varepsilon}})^{1-\alpha}$. If one rearranges the equation to find $\frac{k}{\overline{\varepsilon}} = (\frac{r(\mu)+\delta_d}{\alpha z})^{\frac{1}{\alpha-1}}$ and substitutes $\frac{k}{\overline{\varepsilon}}$ in the wage equation, the wage rate becomes a function of productivity and interest rate, $w(z,\mu) = (1-\alpha)z(\frac{r(\mu)+\delta_d}{\alpha z})^{\frac{\alpha}{\alpha-1}}$. From the equation it is clear that the wage rate increases as productivity increases.

At the end of the period some workers leave islands by choice or exogenously and new workers arrive from other islands. Also, islands may experience a technology shock, which occurs with Poisson frequency δ_z. If an island is hit by the shock, it draws new productivity from a bounded Pareto distribution. Firms start each period with a new set of workers and new productivity if a shock occurs.

Let $L(z)$ be the firms' decision rule for labor and let $K(z)$ be the decision rule for capital rental.

3.3. Government

The government levies income taxes and an estate tax to fund pension, unemployment benefits, and government spending, G. Its budget is balanced each period:

$$\begin{aligned} &\int_s \tau_c(T(w(z;\mu)\varepsilon))ra\mu_h(\varepsilon,a,z)d[\varepsilon\times a\times z]+ \\ &\int_{s_w\in s} T(w(z;\mu)\varepsilon)\mu_h(\varepsilon,a,z)d[\varepsilon\times a\times z]+\int_{s_d\in s}\tau_E(a)\mu_h(\varepsilon,a,z)d[\varepsilon\times a\times z] \\ &\geq \int_{s_r\in s}\tau_r\mu_h(\varepsilon,a,z)d[\varepsilon\times a\times z]+\int_{s_0\in s} b\mu_h(\varepsilon,a,z)d[\varepsilon\times a\times z]+G \end{aligned}$$

where s_w indicates a subset of space where workers are, $(\varepsilon_i\times a\times z_j)$ such that $\varepsilon_i<\varepsilon_{N_\varepsilon}$ and $z_j>0$. s_d indicates a subset of space where newborns are. s_r indicates a subset of space where retirees are – namely, $(\varepsilon_i\times a\times z)$ such that $\varepsilon_i=\varepsilon_{N_\varepsilon}$. Lastly, s_0 is a subset of space for the unemployed, $(\varepsilon_i \times a \times z_j)$ such that $\varepsilon_i < \varepsilon_{N_\varepsilon}$, $z_j = 0$.

3.4. Equilibrium

An equilibrium is a set of functions

$$(r, w, V, W, L, K, \chi, A, G)$$

that solves workers' and firms' problems and clear markets for labor, assets and goods. The government budget is balanced.

1. W solves (1), (S, A) are the associated decision rules

2. V solves (2), (L, K) are the associated decision rules

3. The labor market clears on each island:

   $\int_{z_i \in z} L(z)\mu_f(z)dz = \int_{s_i \in s} \varepsilon\mu_h(\varepsilon, a, z)d[\varepsilon \times a \times z]$, $\forall i = 1, ..., N_z$

4. The capital market clears:

   $\int_z K(z)\mu_f(z)dz = \int_s A(\varepsilon, a, z)\mu_h(\varepsilon, a, z)d[\varepsilon \times a \times z]$

5. The government budget is balanced:

   $\int_s \tau_c(T(w(z;\mu)\varepsilon))ra\mu_h(\varepsilon, a, z)d[\varepsilon \times a \times z] +$

   $\int_{s_w \in s} T(w(z;\mu)\varepsilon)\mu_h(\varepsilon, a, z)d[\varepsilon \times a \times z] + \int_{s_d \in s} \tau_E(a)\mu_h(\varepsilon, a, z)d[\varepsilon \times a \times z]$

   $\geq \int_{s_r \in s} \tau_r\mu_h(\varepsilon, a, z)d[\varepsilon \times a \times z] + \int_{s_0 \in s} b\mu_h(\varepsilon, a, z)d[\varepsilon \times a \times z] + G$

6. $\mu_h(\varepsilon_j, B, z_k) = \int_{\{(\varepsilon_i, a, z_l)|A(\varepsilon_i, a, z_l) \in B\}} \pi(z_k|\varepsilon_i)\pi_{ij}\chi(\varepsilon_i, a, z_l)\mu_h(\varepsilon_i, a, z_l)d[\varepsilon \times a \times z]$

$(1-\delta)\int_{\{(\varepsilon_i,a,z_l)|A(\varepsilon_i,a,z_l)\in B\}} \pi_{lk}\pi_{ij}(1-\chi(\varepsilon_i,a,z_l))\mu_h(\varepsilon_i,a,z_l)d[\varepsilon\times a\times z]+$

$\delta\int_{\{(\varepsilon_i,a,z_l)|A(\varepsilon_i,a,z_l)\in B\}} \pi(z_k|\varepsilon_i)\pi_{ij}(1-\chi(\varepsilon_i,a,z_l))\mu_h(\varepsilon_i,a,z_l)d[\varepsilon\times a\times z]$

$\forall\,(\varepsilon_j,B,z_k)\in\mathbf{S}$

where $a\in\mathbf{B}\subset\mathbf{R}$, $\varepsilon\in\mathbf{E}\equiv\{\varepsilon_1,..,\varepsilon_{n_\varepsilon}\}$, $z\in\mathbf{Z}\equiv\{z_1,..,z_{n_z}\}$, $\mathbf{S}=\mathbf{E}\times\mathbf{B}\times\mathbf{Z}$

7. $\mu_f(z_k)=\int_{\{z_l\in Z\}}\pi_{lk}\mu_f(z_l)dz$, $\forall\,z_k\in\mathbf{Z}$ where $z\in\mathbf{Z}\equiv\{z_1,..,z_{n_z}\}$

4. Calibration

The model period is a year. I divide parameters into two groups: parameters in the first group are exogenously set using data, and the rest of the parameters are jointly calibrated. I calibrate the model targeting data circa 1992 and 2013. Many parameters are common for both periods, but parameters that define the labor productivity process and the firm productivity process, parameters that mainly affect worker allocation over islands, and some parameters that are tax related vary. In the next section, I will compare the realized changes between 1992 and 2013 to the changes between 1992 and the hypothetical economies: (1) an economy without changes in the labor productivity process, (2) an economy without changes in the firm productivity process, and (3) an economy without changes in the parameters that affect worker allocation over islands. This approach allows me to assess the extent to which the realized changes are primarily attributable to the changes in the worker distribution or the firm distribution, or the matching between workers and firms.

4.1. Parameters set ex ante

Utility function and technology I choose log utility function, $u(c) = log(c)$. Production technology is CRS and the capital share is set as residual of labor share of output in Giandrea and Sprague (2017). They measure labor's share[5] of output in the non-farm business sector from 1947 through 2016. The labor share gradually decreases over the estimation period and the trend becomes more pronounced since early 2000. In my model, varying labor share does not affect earnings distribution much because it applies to all workers. Since my interest is the changes in earnings distribution, I use the long-term average (1989 to 2013) of labor share, 60.5%; thus, α is set to 0.395.

Separation rate Unemployment is taken care of separately, so I use job-to-job flow in the LEHD[6] to set the separation rate and the target for the number of searchers. Average job-to-job flow in a quarter from 2000 to 2001[7] is 4.3%. Assuming a worker experiences job transition once a

[5] Sum of employee compensation and proprietors' labor compensation
[6] Job-to-Job Hires (Continuous Employment) (EEHire)
[7] The publicly available data start in 2000.

year on average, job-to-job flow is 17.1% in a year. From JOLTS, layoffs are 45.7% of all separations (2001 to 2016 average), and I pinned down the separation rate as 45.7% of total job-to-job flows, so the value of δ for 1992 is 7.8%. The rest of the flows (9.3%) will be filled with job searchers. Average job-to-job flow from 2011 to 2013 is 3.0%, the separation rate for 2013 is 5.5%, and the target for a job searcher is 6.6% by the same fashion.

Unemployment The model period is a year, but most unemployment is short term. Average duration is 20.1 weeks and median duration is 10.3 weeks from 1990 to 2013 in the Bureau of Labor Statistics data. Hence I target long-term non-employment statistics in Song and Von Wachter (2014). They generate a new long non-employment duration using longitudinal administrative data from the SSA. They measure individuals in the labor force with non-employment spells of 1 to 2 years and 2 to 3 years. I compute the average spell as a weighted average of one year (spells of 1 to 2 years) and two years (spells of 2 to 3 years) from 1990 to 2011. I set p_{em} to 0.7042 to match the average spell, 1.42 years. The probability of becoming unemployed, p_u is pinned down to match the share of the long-term non-employed (11%, the average from 1990 to 2011) in the data. In stationary equilibrium the following equation should hold:

$$(1 - p_{em})ltne + p_u(1 - ltne) = ltne$$

where $ltne$ is long-term non-employment. With $p_{em} = 0.7042$ and $ltne = 11\%$, p_u is 0.0902.

Life cycle Retirement probability, p_d, is set to 2.2% which implies that the average working life is 45 years (American Community Survey, average of 2005 to 2016). The probability of dying is 10.29% to match the share of those 65 years old or older over the number of people who are older than 20 years, 17.6%. The share of those 65 years or over is 17.6% in the 1990 Census, 17.4% in the 2000 Census, and 17.8% in the 2010 Census.[8]

Taxes[9] The parameter that governs tax progressivity, γ, is set to 0.181, following Heathcote, Storesletten, and Violante (2017). They use PSID data (2000 to 2006) and NBER's TAXSIM program to estimate γ. τ_{max} and τ_{min} are set from IRS Statistics of Income Table 23, U.S. Individual Income Tax: Personal Exemptions and Lowest and Highest Bracket Tax Rates, and Tax Base for Regular Tax. From 1990 to 2013, the lowest income tax rate is 10% to 15%, and the highest rate is 28% to 39.6%. Since the share of earnings in the Survey of Consumer Finances (SCF)

[8] Although the share is stable, the number of old population has increased. In 1990, the number of those 65 years and over was about 31 million, and in 2010 it was around 40 million.

[9] Parameters that govern tax and transfer are fixed over the two periods in my model but it is known that tax progressivity has dropped in the U.S. Hubmer, Krusell, and Smith Jr (2016) show the drop in tax progressivity is an important driver of the rise in wealth inequality.

(Table 1) is computed from pretax earnings and the variance of earnings in Song et al. (2015) is computed from pretax earnings[10] as well, the tax rate would not much affect the earnings distribution.[11] It affects to wealth distribution, and I set the minimum and maximum rates to 10% and 39.6%, respectively, to be conservative.

I assume that the capital income tax rate depends on the labor income tax rate as in the U.S. tax system.[12] Tax rates on capital gain are 5% to 20% between 2000 and 2006 in the data, and I set $\tau_c = 0.1$ if the labor income tax is $\tau_{min} w(z;\mu)\varepsilon$ and $\tau_c = 0.2$ otherwise.

I use the estate tax as in Castaneda et al. (2003):

$$\begin{cases} \tau_E(x) = 0, & \text{if } x \leq \underline{x} \\ \tau_E(x) = \tau_E(x - \underline{x}), & \text{if } x > \underline{x} \end{cases}$$

To set $\underline{x}$, I use the exclusion amount for the estate tax from the IRS and per capita GDP from 2001 to 2006.[13] The exclusion amount/per capita GDP ratio is 30.4 (the average between 2001 and 2006), and I set $\underline{x}$ to 30.4 times output per capita in the model. To pin down the estate

[10] Precisely, their measure of earnings is the amount in the Box in the W-2 form. It is total taxable wages, tips, prizes and other compensation, as well as any taxable fringe benefits.

[11] It may affect earnings distribution through workers' searching decision. Since workers have to pay the cost to search, wealthy workers tend to search given a labor productivity, and the tax affects the wealth distribution.

[12] A weakness of this approach is that the only capital income is interest rate income in the model, and most interest income is taxable as ordinary income along with labor income. The rationale of this assumption is the following. There is only single asset in the model that captures all forms of asset of the households, but the interest rate bearing asset is only a little fraction of households' portfolio in the data (the share of bonds is 1.6% in the 2013 SCF, Kuhn and Ríos-Rull (2016)). Considering that the capital income tax rate is lower than the ordinary income tax rate, applying the same tax rate to labor and interest rate income would result in levying a higher tax to most of the households in the model.

[13] The exclusion amount in 2001 is $675,000, and it increased to $2,000,000 in 2006.

tax rate τ_E, I use Estate Tax Statistics Filing Year Table 1 from the IRS. The average net estate tax/taxable estate ratio between 2001 and 2006 is 20% so I set τ_E to 0.2.

4.2. Calibrated parameter

Parameters in this group are jointly calibrated to match moments from various sets of the U.S. data with corresponding steady state moments that are obtained from the model solutions. I describe targets with certain parameters, but since the parameters are jointly determined this association is heuristic.

Preference and depreciation rate I set the household discount factor β to 0.9521 to match the long term average (1983 to 1992) real interest rate around 1992, 3.79%. I use the 1-year U.S. Treasury bill rate and personal consumption expenditures excluding food and energy to compute the real interest rate. Depreciation rate δ_d is set to 0.0884 to match the capital-output ratio 3.13. The target value 3.13 is taken from Castaneda et al. (2003); their measure of capital is average household wealth in the 1992 SCF ($184,000) and output is the U.S. per household GDP in 1992 ($58,916). The equilibrium interest of the model is 3.8% (1992), and the capital output ratio is 3.13. The number of parameters is 2, and the number of targets is 2.

Labor productivity Parameters that are related to the labor productivity process are chosen to match the earnings distribution. I target the earnings Gini index, shares of earnings by quintile, and the top 10% (top 90% to 95%, 95% to 99%, top 1%) from the SCF 1992 and 2013.[14] I assume a worker's productivity is drawn from a bounded Pareto distribution. With probability δ_ε, workers lose their current labor productivity and draw a new one from the distribution. The lower bound ($\underline{\varepsilon}_1$), upper bound ($\overline{\varepsilon}_1$), and shape parameter ($\eta_{\varepsilon,1}$) for 1992 are set to 0.3, 14.0, and 1.94, respectively. For 2013, $\underline{\varepsilon}_2$, $\overline{\varepsilon}_2$, and $\eta_{\varepsilon,2}$ are 0.3, 29.0, and 1.81, respectively. Shock probability, δ_ε, is set to 0.05 for both periods. I set the number of grid points to 7, and the 7th point is assigned to the retirees; hence, $\varepsilon_7 = 0$. The 1st to the 6th values are set to represent the [10.0, 31.0, 28.0, 28.5, 2.45, 0.5] (%) of the working population, respectively.[15] For example, ε_2 is a median value between x_1 and x_2 such that $f(x_1) = \frac{1-(\underline{\varepsilon}_1/x_1)^{\eta_\varepsilon}}{1-(\underline{\varepsilon}_1/\overline{\varepsilon}_1)^{\eta_\varepsilon}} = 0.1$, $f(x_2) = \frac{1-(\underline{\varepsilon}_1/x_2)^{\eta_\varepsilon}}{1-(\underline{\varepsilon}_1/\overline{\varepsilon}_1)^{\eta_\varepsilon}} = 0.1 + 0.31$ where $f(x_i)$ is the CDF of the bounded Pareto distribution.

Figure 7 shows the discretized labor productivity values for both periods. From 1992 to 2013, the value of the top 2 points increased while the value of the other 4 points remained relatively stable. In 1992, the value of the 5th and 6th grid points are 19.7, and 39.2 times higher, respectively, than the 1st point's value, and in 2013 they are 29.5 and

[14] Statistics in the table are taken from Castaneda et al. (2003) and Kuhn and Ríos-Rull (2016).

[15] The share from the 1st to the 3rd bin are chosen to represent the population with education attainment levels: less than a high school diploma, high school graduates, some college (average of 1992 to 2013, BLS). The rest of the population has a bachelor's degree or higher, and I add two points to capture earnings concentration to the top.

72.7 time higher, respectively, than the 1st point's value. I do not take a stance on what has caused the change in the labor productivity process. It could be a combination of the changes in the workers' skill distribution, the price of those skills and working hours as portrayed in Heathcote, Storesletten, and Violante (2010). Since the focus of this paper is decomposing the factors affecting earnings distribution into worker and firm components, I wrap the changes in the worker side to their labor productivity process.

Table 1: Earnings distribution

	Gini	**Quintiles**					**Top(%)**		
		1q	2q	3q	4q	5q	90-95	95-99	99-100
1992									
Data	0.63	-0.40	3.19	12.49	23.33	61.39	12.38	16.37	14.76
Model	0.64	0.00	3.49	10.90	17.81	67.80	13.77	16.61	13.50
2013									
Data	0.67	-0.10	3.00	10.40	20.20	66.50	12.40	18.40	18.80
Model	0.68	0.00	3.00	8.82	15.68	72.50	12.58	19.21	16.76

Data: SCF

Table 1 shows the targets and the model results. The model does a good job of resembling the overall distributions. The number of parameters is 8, and the number of targets is 18.

Firm productivity, job transition probability and search cost The parameters that define the firm productivity process, the job transition probability over islands and the search cost are pinned down to match the variance decomposition results in Song et al. (2015) and firm size distribution (employment) in the Business Dynamics Statistics (BDS).

I assume an island's productivity is drawn from the bounded Pareto distribution. As in the labor productivity process, with probability δ_z, islands lose the current productivity and draw a new one. I set the lower bound ($\underline{z}_1$), the upper bound ($\overline{z}_1$), and the shape parameter (η_1) for 1992 to 0.5, 1.3, and 4.2, respectively. For 2013, they ($\underline{z}_2$, $\overline{z}_2$, η_2) are 0.53, 1.3, and 4.6, respectively. I discretize the firm productivity into 10 grid points, so the number of islands in the economy is 10. The 1st to the 10th values are set to represent [48.0, 32.0, 6.0, 4.0, 3.0, 2.0, 2.0, 1.6, 0.4, 0.1] (%) of total firms in the economy, respectively.

Song et al. (2015) use the U.S. SSA data, which connect employees to employers. First they decompose individual earnings into person and firm components using the regression model introduced by Abowd, Kramarz, and Margolis (1999):

$$y_t^{i,j} = \nu^i + \psi^j + X_t^i \xi + e_t^{i,j}$$

where $y_t^{i,j}$ is a real log earnings of worker i at firm j at time t. ν^i captures fixed worker characteristics such as innate ability, and ψ^j captures firm component in earnings (e.g. compensation differentials). ξ captures time varying worker characteristics such as experience, and $e_t^{i,j}$ captures transitory earnings fluctuation. Ignoring time-varying component, $X_t^i = 0$ and rewrite $y_t^{i,j}$ as

$$y_t^{i,j} = (\nu^i - \overline{\nu}^j) + \overline{\nu}^j + \psi^j + e_t^{i,j}$$

From the above equation, they define variances that depend on firm j component as *between firm* components and the rest of the part as *within firm* components:

$$var(y_t^{i,j}) = \underbrace{var(\overline{\nu}^j) + var(\psi^j) + cov(\overline{\nu}^j, \psi^j)}_{\text{between-firm components}} + \underbrace{var(\nu^i - \overline{\nu}^j) + var(e_t^{i,j})}_{\text{within firm components}}$$

In the model, a worker's earning is $w(z, \mu)\varepsilon$. Convert it to log, and it becomes $log(\varepsilon)+log(w(z, \mu))$, so $log(\varepsilon)$ corresponds to ν and ψ corresponds to $log(w(z, \mu))$. I adjust some of the statistics in Song et al. (2015) to set targets. First, since there is no time-varying worker characteristics in the model and yet X is a worker's characteristic in the decomposition, I map the variance of $\nu + X\xi$ to the variance of ε in the model. Second, I subtract $var(e)$ from the data since there is no transitory earnings shock in the model.

Song et al. (2015) repeated the variance decomposition for five adjacent seven-year intervals from 1980 to 2013, and I target results using 1987-93 and 2007-13 data. One thing worth mentioning is that Song et al.'s (2015) variance decomposition used the sample that drops the small firms – firms that hires less than 20 workers. To be consistent with the targets, I dropped the small firms when computing the moments using the results of the model as well. Figure 8 shows wage rates by firm productivity, given the equilibrium interest rates of the corresponding periods.[16] Unlike labor productivity, firm productivity has become less dispersed in 2013. It implies that the wage premium of being employed at a high-productivity firm has decreased. Qualitatively, this change is consistent with the evidence reported in Bloom, Guvenen, Smith, Song, and von Wachter (2018).

Table 2 presents the targets and the model results. For variance of earnings as well as its components from the decomposition, the difference between 1992 and 2013 is fairly close to the data.

Table 2: Variance of earnings

	Earnings	**Within**	**Between**				ν
		$\nu^i - \overline{\nu}^j$	$\overline{\nu}$ + ψ+$2cov(\overline{\nu}, \psi)$	$\overline{\nu}$	ψ	$2cov(\overline{\nu}, \psi)$	
1992							
Data	0.63	0.37	0.25	0.10	0.08	0.08	0.47
Model	0.66	0.42	0.24	0.04	0.08	0.11	0.46
2013							
Data	0.79	0.42	0.37	0.16	0.08	0.14	0.57
Model	0.82	0.45	0.36	0.12	0.08	0.17	0.57
2013-1992							
Data	0.16	0.05	0.12	0.06	0.00	0.06	0.10
Model	0.16	0.03	0.12	0.08	0.00	0.06	0.11

* 1992: results using 1987-93 data; 2013: 2007-13 data
** Data values are adjusted to be consistent with the model. First, $var(e)$ (1992: 0.15, 2013: 0.14) is subtracted from $var(y)$ and $var(\nu^i - \overline{\nu}^j)$. Second, $var(X\xi)$ is added to $var(\nu)$, $var(X\xi - \overline{X\xi})$ and $2cov(\nu - \overline{\nu}, X\xi - \overline{X\xi})$ are added to $var(\nu^i - \overline{\nu}^j)$.
Data: Song et al. (2015)

If a worker decides to search or is exogenously separated from a firm, she will move to an adjacent island following the job transition probability. I use Beta distribution to set the transition probability

[16] Given an interest rate, there is a one-to-one mapping between a z and a wage rate.

over the islands. The island distribution is mapped to a support of a Beta CDF, $F_x(a,b)$, and the probability of moving to ith island is simply $F_{x_i}(a,b) - F_{x_{i-1}}(a,b)$ where x_i is the cumulative measure of islands up to the ith point. Transition probabilities depend on worker types, and $\{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$, $\{\varepsilon_4, \varepsilon_5, \varepsilon_6\}$ follows the same transition process. As labor productivity increases, a worker has a higher chance to arrive at a high-productivity island and vice versa.[17] The shape parameters for Beta distributions are listed in Table 8, where $a_{i,j}$ is a shape parameter for ith worker group in period j.[18]

The job transition process and the search cost mainly shape the firm size distribution. This is different from models with heterogeneous firms that assume decreasing return to scale technology to have non degenerate firm distribution (for example, Khan and Thomas (2008), Khan and Thomas (2013)). Those models typically assume homogeneous workers and a frictionless labor market, so firms hire workers where a wage rate equals a marginal labor productivity. Given a wage rate and capital, the size of employment becomes a function of a firm's productivity and it is increasing in the productivity. In my model, a firm also hires workers where a marginal labor productivity equals a wage rate. However, given the CRS technology assumption and the frictionless capital market, it is optimal to hire all available workers on an

[17] In other words, a transition probability associated with a high labor productivity has a first-order stochastic dominance over a process with a low productivity.

[18] a is inversely related to the mass on the left side and b is inversely related to the mass on the right side.

island as long as workers are willing to accept the wage rate. Therefore, a firm size distribution is determined by how many workers arrive to an island, which is controlled by the job transition probability and how many workers leave by searching and separation.

Thanks to the search option, once a worker moves to a low-productivity island, she may leave. Thus, some islands end up with less workers, and some islands hire more workers than the job transition probability implies in a stationary equilibrium. The number of searchers is mainly affected by the search cost. Table 3 shows firm size distribution and the share of workers who search over the total working population. Firm size distribution is the employment share of firms that hire n workers. The target for 1992 is the average of 1990 to 1992, and target for 2013 is the average of 2011 to 2013 in the BDS. The difference in firm size distribution between 1992 and 2013 is not large, but the employment share of firms that hire more than 250 workers increased, and the employment share of small firms fell. The model captures this shift closely as well as matches the share of searchers.[19]

The number of parameters is 20 and the number of targets is 24 in this section.

Taxes and transfers τ_i is set to 0.83 and 0.82 for 1992 and 2013, respectively, to match the average marginal tax rate of 34% (Heathcote

[19] Targets for searcher is the fraction of job to job flows which is explained in the separation rate section.

Table 3: Firm size distribution* and worker flow

	Firm size distribution						Searcher
	1-4	5-19	20-49	50-249	250-4999	5000+	(%)
1992							
Data	5.8	14.9	10.8	16.0	22.9	29.5	9.3
Model	5.9	9.8	14.2	18.1	22.2	29.7	9.6
2013							
Data	5.2	12.9	9.8	15.3	23.7	33.2	6.6
Model	5.6	10.2	9.8	20.3	20.3	33.8	6.6

* Share of employment of firms who hire n workers
Data: BDS

et al. (2017)). Transfer to retirees, τ_r and to unemployed, b, are set to 0.5 and 0.049 for 1992 and 0.47 and 0.047 for 2013 to match transfer to retirees output ratio and the unemployment benefit output ratio. To set these targets, I use the data in the Social Security Administration Annual Statistical Supplement to the Social Security Bulletin. Transfer to retirees is the sum of Old-Age Survivors Insurance and Medicare (Table 4A, Table 8A). Transfer to unemployed is Benefits Paid in Unemployment Insurance (Table 9A). Output is GDP, and the targets are 5.9% and 0.3%, which are the average ratios between 2000 and 2006. The number of targets is 6 and the number of parameters is 6. Table 4 shows the targets and model values.

Table 4: Targets and model values for taxes and transfers

	Target	Model	
		1992	2013
Average marginal tax rate	34.0	34.2	34.6
Transfer(retiree) / output	5.9	5.9	5.5
Transfer(unemployed) / output	0.3	0.3	0.3

Data: SSA, Heathcote et al. (2017)

Table 5: Wealth distribution

	Gini	Quintiles					Top(%)		
		1q	2q	3q	4q	5q	90-95	95-99	99-100
1992									
Data	0.78	-0.39	1.74	5.72	13.43	79.49	12.62	23.95	29.55
Model	0.71	-0.69	2.59	7.79	18.91	71.40	13.84	25.32	14.71
2013									
Data	0.85	-0.70	0.60	3.20	9.80	87.00	12.10	27.40	35.50
Model	0.78	-1.32	0.97	5.53	16.37	78.44	13.88	30.04	18.69

Data: SCF

Table 6: Earnings dynamics

	Std.		**Skewness**		**Kurtosis**		**P($\|\Delta y\|$) $<$ x***		
	1y	5y	1y	5y	1y	5y	x = 0.2	0.5	1.0
Data	0.51	0.78	-1.07	-1.25	14.93	9.51	0.67	0.83	0.93
Model									
1992	0.47	0.72	-0.04	0.15	9.26	6.51	0.78	0.83	0.90
2013	0.43	0.73	-0.03	0.31	12.94	8.40	0.81	0.86	0.92

* $|\Delta y|$: Absolute log earnings change less than a threshold x
Data: Guvenen et al. (2015)

4.3. Validation

I compute non-targeted moments to check the model's validity. First, the model matches the wealth distribution quite well except at the top 1%. It misses the levels but captures the significant fraction of the rise in the concentration between 1992 and 2013 which implies that changes in the labor income distribution could be one of the main drivers of the rise in wealth inequality.

Second, the model generates features of earnings dynamics reasonably well. Using SSA data, Guvenen et al. (2015) compute various moments of the U.S. earnings dynamics.[20] They show that earnings dynamics are very different from a log normal process, which has been the standard assumption in the incomplete market literature. I simulate the model[21] to compute the same moments. Table 6 shows moments from the data and the model. My model does a fairly good job of generating moments that are consistent with the data except the skewness.

5. Results

Section 4 demonstrates the ability of the model to capture the earnings distribution, variance of earnings and its components, and the firm distribution. This section explores the sources and implications of rising inequality. First, Sections 5.1 and 5.2 describe the results without the search option or the firm heterogeneity to show how the model works.

5.1. The Role of Search

To see the role of the search option, I remove the option and solve the model with parameters capturing the 1992 economy. Without searching, firm size distribution and worker composition within firms, which

[20] Moments are computed using 1994-2013 data.

[21] The panel size is 2000 and the simulation period is 1200. The 1st through the 200th periods simulated series are discarded when computing the statistics. Increasing the panel size or the number of periods has little effect on the results.

Table 7: Parameter values

Parameter	Value	Description
Preferences and production		
β	0.9521	Discount rate
α	0.395	Capital share
δ_d	0.0884	Depreciation rate
Life cycle		
p_r	0.022	Retirement probability
p_d	0.1029	Dying probability
Tax and transfer		
τ_{min}	0.1	Minimum tax rate
τ_{mix}	0.396	Maximum tax rate
τ_{c1}	0.1	Capital income tax rate
τ_{c2}	0.2	Capital income tax rate
γ	0.181	Tax progressivity
$\tau_{i,1}$	0.83	Labor income tax
$\tau_{i,2}$	0.82	Labor income tax
$\tau_{r,1}$	0.5	Transfer to retirees
$\tau_{r,2}$	0.47	Transfer to retirees
b_1	0.049	Transfer to unemployed
b_2	0.047	Transfer to unemployed
Unemployment		
p_u	0.0902	Unemployment shock probability
p_{em}	0.7042	Employment shock probability
Labor productivity		
$\underline{\varepsilon}_1$	0.3	Lower bound of Pareto distribution
$\overline{\varepsilon}_1$	14.0	Upper bound of Pareto distribution
$\eta_{\varepsilon,1}$	1.94	Shape of Pareto distribution
$\underline{\varepsilon}_2$	0.3	Lower bound of Pareto distribution
$\overline{\varepsilon}_2$	29.0	Upper bound of Pareto distribution
$\eta_{\varepsilon,2}$	1.81	Shape of Pareto distribution
δ_ε	0.05	Shock probability

Table 8: Parameter values

Parameter	Value	Description
Firm producitivity		
$\underline{z}_1$	0.5	Lower bound of Pareto distribution
$\overline{z}_1$	1.3	Upper bound of Pareto distribution
η_1	4.2	Shape of Pareto distribution
$\underline{z}_2$	0.53	Lower bound of Pareto distribution
$\overline{z}_2$	1.3	Upper bound of Pareto distribution
η_2	4.6	Shape of Pareto distribution
δ_z	0.08	Shock probability
Job transition		
$a_{1,1}$	1.5	Shape of Beta distribution
$b_{1,1}$	0.26	Shape of Beta distribution
$a_{2,1}$	3.5	Shape of Beta distribution
$b_{2,1}$	0.165	Shape of Beta distribution
$a_{1,2}$	2.7	Shape of Beta distribution
$b_{1,2}$	0.32	Shape of Beta distribution
$a_{2,2}$	14.2	Shape of Beta distribution
$b_{2,2}$	0.11	Shape of Beta distribution
s_1	0.51	Search cost
s_2	0.53	Search cost
δ_1	0.0783	Separation rate
δ_2	0.0556	Separation rate

Table 9: Firm size distribution*

	Firm size distribution					
	1-4	5-19	20-49	50-249	250-4999	5000+
1992						
Searching	5.9	9.8	14.2	18.1	22.2	29.7
No searching	17.4	33.3	15.3	14.9	19.1	0.0

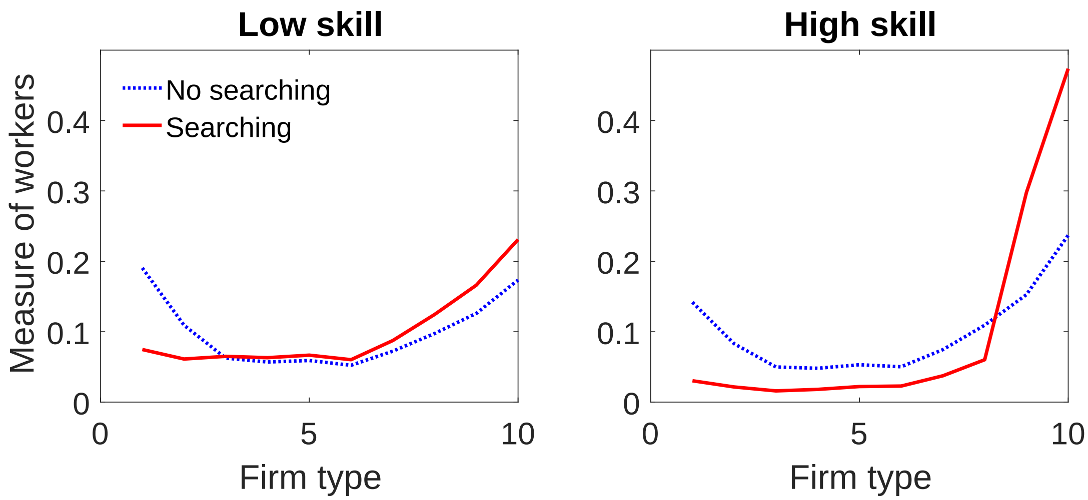
* Share of employment of firms who hire n workers

affect earnings distribution as well as earnings variance, change. Table 9 shows the firm size distribution with and without the search option. First, the firm size distribution becomes relatively flat. With the search option, some workers leave low-wage firms, but no one leaves high-wage firms. Since this is not possible without the search option, there are many small to mid-size firms, and there is no very large firm. Also, the composition of workers within firms will be different.

With the search option, positive sorting arises. In the model, productive workers are more likely to search. First of all, productive workers tend to be rich and, hence, can afford the search cost. Also, the expected value of searching is higher for them. Recall that the job transition probability depends on ε in a way that productive workers have a better chance to move to high-wage firms. Moreover, once productive workers move to a high-wage firm, their earnings increase more than the rise in earnings of low-productivity workers.[22] Thus, productive workers engage in searching, and have a higher chance to move up, while low-productive workers tend to stay where they are. Figure 1 shows

[22] Since earnings is $w(z;\mu)\varepsilon$, the benefit of having high $w(z;\mu)$ is large when ε is high.

Figure 1. Worker allocations over islands



Low skill shows workers who have ε_1 and high skill shows workers with ε_6. Measures of each type are normalized to 1.

worker allocations over firms with and without the search option. It is clear that productive workers are more likely to be sorted into high-wage firms by searching.

Variance of earnings changes without the search option as well, decreasing from 0.66 to 0.54. Within-firm variance increases, since worker compositions within firms are not so different from the overall worker distribution. It is not the same for all islands due to type-dependent transition probability, $\pi(z|\varepsilon)$, but the sorting induced by $\pi(z|\varepsilon)$ is not large: $2cov(\overline{\nu}, \psi)$ decreases to 0.02 without the search option. Also, $var(\overline{\nu})$ becomes close to zero, indicating there is almost no segregation.

Unlike the variance of earnings, the earnings distribution does not change much. The share of earnings of the 5th quintile decreases a little. Interestingly, the earnings share of the top 1% and the top 10%

Table 10: Variance of earnings

	Earnings	**Within**	**Between**				ν
		$\nu^i - \overline{\nu}^j$	$\overline{\nu} + \psi + 2cov(\overline{\nu}, \psi)$	$\overline{\nu}$	ψ	$2cov(\overline{\nu}, \psi)$	
Searching	0.66	0.42	0.24	0.04	0.08	0.11	0.46
No searching	0.54	0.47	0.06	0.01	0.03	0.02	0.48

Table 11: Earnings distribution

	Gini	**Quintiles**					**Top(%)**		
		1q	2q	3q	4q	5q	90-95	95-99	99-100
Searching	0.64	0.00	3.49	10.90	17.81	67.80	13.77	16.61	13.50
No searching	0.65	0.00	3.60	10.12	19.32	66.96	16.61	16.08	15.58

rise. Perhaps it is obvious, but the degrees of sorting and segregation are not related to the degrees of earnings concentration. Without the search option, there are less workers at high-wage firms; hence, the total earnings decreases. Therefore, the share of earnings of productive workers who happen to be hired by high-wage firms increases.

5.2. The Role of Heterogeneous Firms

In this section I collapse the firm distribution to see the role of heterogeneous firms. I assume all workers are hired by the most productive firm in the 1992 economy.[23] Since the model is calibrated to match vari-

[23] I have tried the case that all workers are hired by a firm with average productivity ($\int_z z\mu_f(z)dz$) in the 1992 economy. It will soon be shown, that workers save less in a homogeneous firm economy. If all workers are hired by the average firm, low savings could have been caused by a lower level of earnings than in the heterogeneous firm economy, since in the latter case the productive firms that pay high wages hire the majority of workers. Indeed, if all workers are hired by a firm with average productivity, more than 40% of the population ends up with negative wealth, and the Gini index rises to 0.77.

ance with two-sided heterogeneity, when differences among firms are collapsed, the variance of earnings that affects earnings distribution decreases.

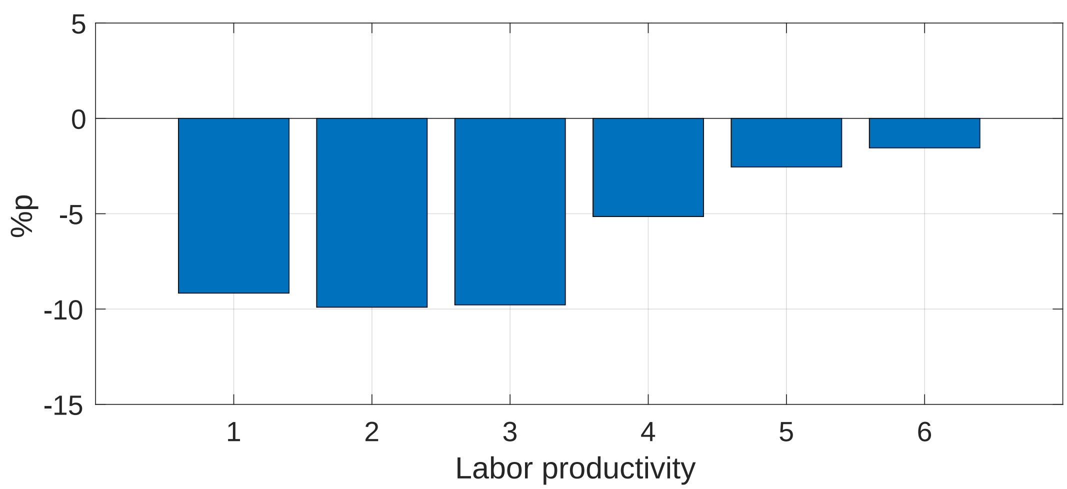
Table 12 shows the earnings and the wealth distribution of the representative firm economy. Without the firm heterogeneity, earnings distribution solely depends on the ergodic distribution of labor productivity process. The variance of earnings decreases to 0.48, and the earnings Gini, the 5th quintile share of earnings and the top 10% share of earnings drop. Interestingly, the wealth Gini index increases while the earnings Gini index decreases. The wealth share of the 1st through 4th quintiles drops, while the wealth share of the 5th quintile and the share of the top 10% increase. Intuitively, when there is no firm heterogeneity, earnings risks from the firm side (the separation, the firm productivity shock) disappear. Therefore, the desire of workers' to save falls. Figure 2 shows the differences in savings rates between the representative firm economy and the heterogeneous firm economy. Quantitatively, agents with low earnings and low wealth decrease their savings more than the high earners and the rich, and wealth inequality rises.[24]

[24] Since capital supply is lower and capital demand is higher (all firms' productivity is the highest possible level of productivity of the heterogeneous firm economy) in the representative firm economy, the equilibrium interest rate (4.5%) is higher than the heterogeneous firm economy (3.8%). If I hold the interest rate at 3.8% in the representative firm economy, wealth inequality rises further. The wealth Gini index rises to 0.79 and the wealth share of the 5th quintile becomes 80.6%.

Table 12: Earnings and wealth distribution

	Gini	Quintiles					Top(%)		
		1q	2q	3q	4q	5q	90-95	95-99	99-100
Earnings									
Het. firm	0.64	0.00	3.49	10.90	17.81	67.80	13.77	16.61	13.50
Rep. firm	0.57	0.00	7.57	12.94	20.04	59.45	10.96	15.68	10.88
Wealth									
Het. firm	0.71	-0.69	2.59	7.79	18.91	71.40	13.84	25.32	14.71
Rep. firm	0.75	-0.68	1.47	5.68	17.90	75.62	15.28	27.24	15.48

Figure 2. Differences in savings rates (Representative firms - Heterogeneous firms)



5.3. *How is the rise in firm inequality related to the rise in earnings inequality?*

Given the assumption that earnings can be decomposed into a worker fixed effect (ε) and a firm fixed effect (z), the changes in earnings distribution are caused by the changes in the ε distribution, z distribution, and worker-firm matchings. To assess the extent to which the realized changes are primarily attributable to the changes the in worker side, or the firm side, or matching between workers and firms, I compare the realized changes between 1992 and 2013 (*Base*) to the changes between 1992 and the hypothetical economies: (1) an economy without changes in the labor productivity process (*Worker*), (2) an economy without changes in firm productivity process (*Firm*), and (3) an economy without changes in the parameters that affect worker allocation over firms (separation rate, search cost, and $\pi(z|\varepsilon)$) (*Matching*).

Table 13: Changes in the firm size distribution*

	Firm size distribution					
	1-4	5-19	20-49	50-249	250-4999	5000+
Base	-0.3	0.5	-4.5	2.2	-2.0	4.1
Worker	-1.1	-1.3	-4.5	3.3	-1.1	4.7
Firm	-0.7	-0.8	-4.7	3.0	-1.4	4.5
Matching	1.5	3.9	2.7	-4.9	-1.6	-1.6

* Share of employment of firms who hire n workers
Base: 2013 minus 1992
Worker: the economy without changes in the labor productivity process - 1992
Firm: the economy without changes in the firm productivity process
Matching: the economy without changes in the search cost, the separation rate, and the job transition process - 1992

Firm size distribution Table 13 presents the changes in the firm size distribution. Not surprisingly, without the changes in the search cost, the separation rate, and $\pi(z|\varepsilon)$, the rise in the firm concentration completely disappears. In fact, there is less concentration to large firms. The share of employment of the largest firms decreases by 1.6%. When shutting off the changes in the labor productivity process and the firm productivity process, the number of searchers is larger than 2013; hence, the share of workers of large firms rises more than the base. In both *Worker* and *Firm*, the number of searchers increases because some low-skill workers who previously chose to stay decided to search. [25]

Since large firms' wage rates are higher, these changes have an impact on the earnings distribution. I now turn to the earnings inequality measures.

Table 14: Changes in the variance of earnings

	Earnings	**Within**	**Between**				ν
		$\nu^i - \overline{\nu}^j$	$\overline{\nu}$ + ψ+$2cov(\overline{\nu}, \psi)$	$\overline{\nu}$	ψ	$2cov(\overline{\nu}, \psi)$	
Base	0.16	0.03	0.12	0.07	0.00	0.06	0.11
Worker	0.03	-0.05	0.08	0.05	-0.01	0.04	0.00
Firm	0.16	0.03	0.13	0.07	0.00	0.06	0.11
Matching	0.13	0.11	0.02	0.01	-0.01	0.01	0.12

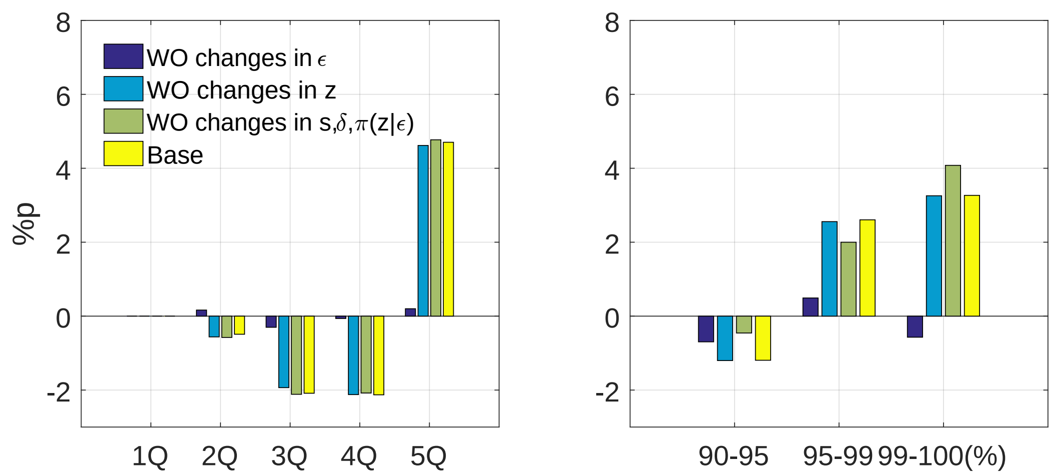
Base: 2013 minus 1992
Worker: the economy without changes in the labor productivity process - 1992
Firm: the economy without changes in the firm productivity process
Matching: the economy without changes in the search cost, the separation rate, and the job transition process - 1992

[25] For example, without changes in the ε process, 3.6% of workers with the lowest labor productivity search, while 1.8% of them search in the 2013 economy. Without the changes in the ε process, the lowest wage is higher, so more workers can afford to search. Without the changes in the z process, workers have a stronger incentive to leave low-wage islands, since z is more dispersed in 1992.

Variance of earnings Table 14 shows the changes in the variance of earnings and its decomposition. The first column reports the changes in the earnings variance. Without the changes in the labor productivity process, the earnings variance only rises 0.03. In contrast, without the changes in the firm productivity process, earnings variance rises 0.16, which is the same amount as the base. When shutting off the changes in the search cost, the separation rate, and $\pi(z|\varepsilon)$, the variance rises 0.13, which is 81% of the rise in the *Base*. Thus, most of the rise in earnings variance is driven by the changes in the labor productivity process. Considering the variance of worker fixed effect (ν) rises 0.11 while the variance of firm fixed effect (ψ) remains at the same level between 1992 and 2013, this is not a surprising result.

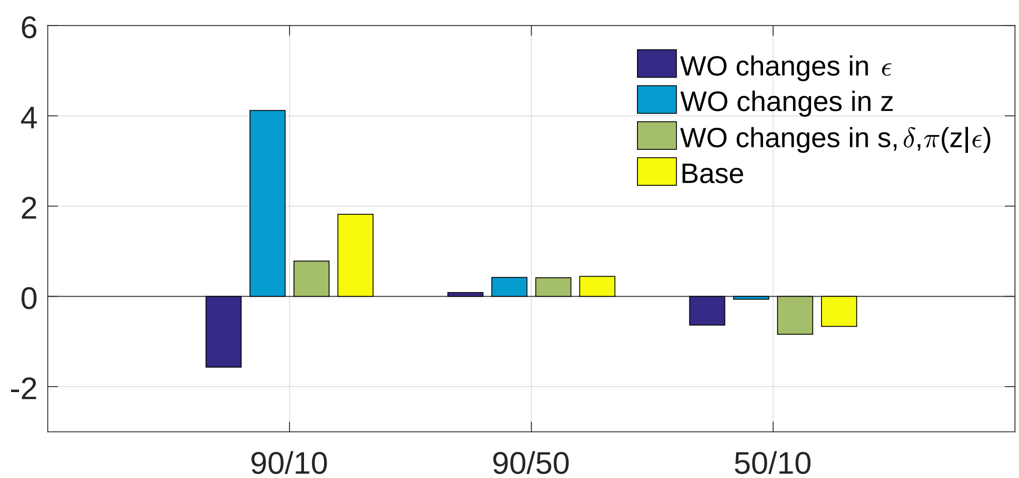
The 2nd through 5th columns show variance decomposition results. A few things are worth mentioning. First, shutting off the changes in the z process shifts the firm size distribution, but it makes almost no change in the variance of earnings. Second, even though the changes in the ε process drive most of the rise in the earnings variance, the 2nd and 4th rows shows that the changes in labor productivity and worker allocation over firms jointly make the changes in the earnings distribution. If the changes in the labor productivity process is the main driver of the changes in the earnings distribution, the rise in the within-firm variance and the between-firm variance should have been rolled back. Instead, the between-firm variance rise 2/3 of the rise in the *Base*, while the within-firm variance decreases by 0.05. Likewise, in *Matching*, the between-firm variance is sharply understated. The rises in segregation ($var(\overline{\nu})$) and sorting ($cov(\overline{\nu},\psi)$) are also abated.

Figure 3. Changes in the earnings distribution



Share of earnings by quintile (left) and top 10% (right)

Figure 4. Changes in the 90/10, 90/50, and 50/10 ratios

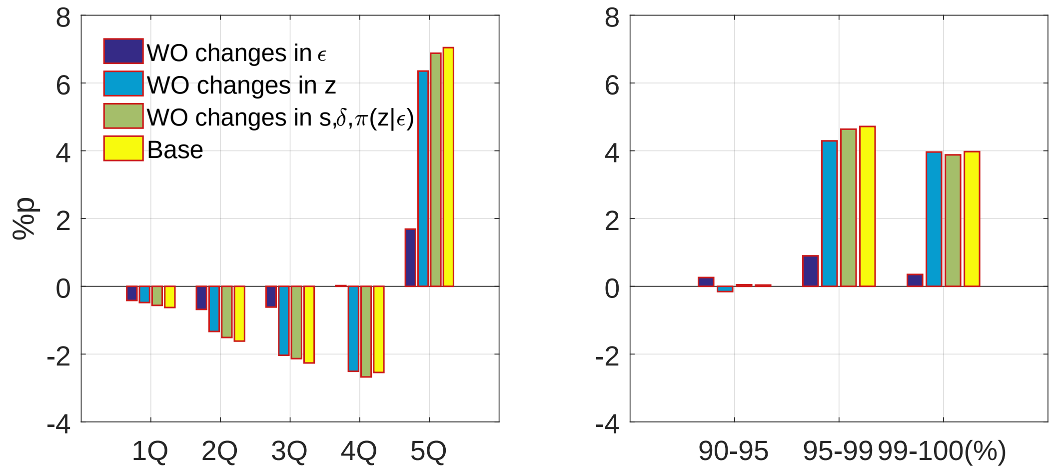


Earnings distribution Figure 3 shows the changes in the earnings share by quintile (left) and by the top 10% (right). It confirms that changes in the labor productivity process are the main driver of the rise in the earnings inequality – without them the rises in earnings by the 5th quintile and the top 10% are greatly understated. The earnings Gini index drops to 0.64, which is the 1992 level.

The changes in the search cost, the separation rate, and $\pi(z|\varepsilon)$ do not increase earnings inequality but mitigate it. When shutting off the changes in them, the share of earnings by the 5th quintile rises 4.7%p which is 101% of the rise in the *Base*. The share of earnings of the top 1% rises 124% of the *Base* and the earnings Gini index is 0.69 which is higher than the 2013 steady state level (0.68). Why is this the case? The reason is similar to the case without the search option. It is shown that there are less workers at large, high-wage firms, and the workers at the large firms are less productive on average without the changes in the search cost, the separation rate, and $\pi(z|\varepsilon)$. Therefore, total earnings fall, and the earnings share of the top 1% – productive workers at high-wage firms – rises.

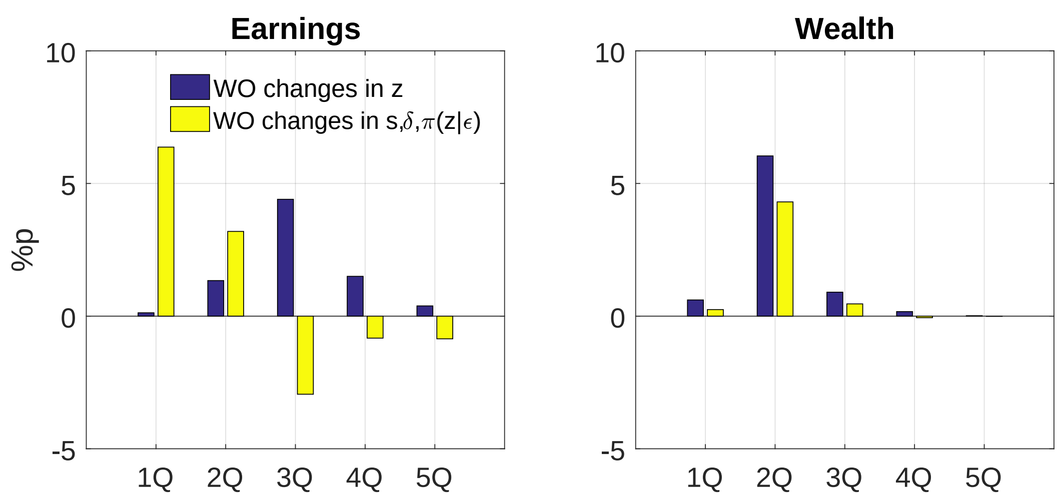
In Figure 3, shutting off the changes in the firm productivity process does not make noticeable differences. Together with the results in Table 14, it seems that the changes in the firm productivity process have little effect on the changes in the earnings distribution. However, Figure 4 reveals that without changes in the firm productivity process, the 90/10 earnings ratio would have risen more, which implies the changes in the firm productivity process also mitigate the rise in earnings inequality.

Figure 5. Changes in the wealth distribution



Share of earnings by quintile (left) and top 10% (right)

Figure 6. Savings rates differences



Saving rates of the economy without changes in z or s, δ, and $\pi(z|\varepsilon)$ - saving rates of 2013
Differences by earnings quintile (left) and wealth quintile (right)

Wealth distribution Although the focus of the exercise is on the changes in earnings inequality over time, it is informative to see the changes in the wealth distribution. Figure 6 presents the changes in the wealth share by quintile (left) and by the top 10% (right). Like in any other measures, the changes in the labor productivity process have the greatest effect, but not as much as to the earnings distribution.

Unlike the changes in the firm productivity process, changes in the search cost, the separation rate, and $\pi(z|\varepsilon)$ mitigate the rise in earnings inequality, these changes increase wealth inequality. Figure 6 shows savings rates differences between the economy without the changes in the firm productivity process and 2013, and savings rates differences between the economy without changes in s, δ, and $\pi(z|\varepsilon)$ and 2013. Without changes in the firm productivity process, people tend to save more. In particular, middle-income workers increase savings more than others, so wealth concentration becomes less severe than in the 2013. Likewise, without the changes in parameters that affect worker allocation over firms, low earnings and poor households' savings rise more than high earnings and wealthy households'. Overall, the higher savings rates in the conterfactuals indicate that the changes in the firm side do not result in higher earnings risks to the households.

Output, capital, and interest rate On top of accounting earnings and firm inequality trends, I explore the implications of these changes

Table 15: Changes in the aggregate variables

	Output (%)	Capital (%)	Interest rate (%p)
Data	68.4	101.3	-2.4
Base	1.3	3.6	-0.3
Worker	-1.1	-1.4	0.0
Firm	7.6	10.1	-0.3
Matching	-4.3	-1.9	-0.3

Data: Output: GDP in BEA, Capital: 'Household Wealth' in Piketty, Saez, and Zucman (2016), Interest rate: Holston, Laubach, and Williams (2017)
Base: 2013 minus 1992
Worker: the economy without changes in the labor productivity process - 1992
Firm: the economy without changes in the firm productivity process
Matching: the economy without changes in the search cost, the separation rate, and the job transition process - 1992

to the aggregate variables. One might be interested in the impact of the rise in sorting, segregation, and firm inequality on aggregate output. As more productive workers increasingly work at productive firms (rise in sorting) and the employment share of large firms increases, total output may increase.[26] The U.S. GDP rose 68.4% between 1992 and 2013, but this may have been the result of technology improvements, population increases, and so on. To tease out the effect of the rise in sorting, segregation, and firm inequality, I keep total effective labor and average firm productivity constant in all equilibria, [27] so if there are changes in aggregates, they come from changes in the distributions of the resources, not from their quantities.

Table 15 presents the changes in the aggregate variables. Between

[26]See Restuccia and Rogerson (2008), Hsieh and Klenow (2009) for the relationship between resource allocation and TFP.

[27] In other words, $\int_{s_i} \varepsilon_i \mu_{hi}(\varepsilon_i, a, z_i) d[\varepsilon_i \times a \times z_i] = \int_{s_j} \varepsilon_j \mu_{hj}(\varepsilon_j, a, z_j) d[\varepsilon_j \times a \times z_j]$, $\int_{z_i} z_i \mu_{fi}(z_i) dz_i = \int_{z_j} z_j \mu_{fj}(z_j) dz_j$ for all i, j.

1992 and 2013, output increases 1.3%, which is small compared to the growth in U.S. GDP (68.4%). Without the changes in the firm productivity process, output increases 7.6%, which is 11% of the rise in the data. Recall the identified changes in the firm productivity process direct to less dispersion of the distribution in 2013. Although productive firms in 2013 hire a larger number of workers as well as more productive workers, their productivities are lower than those of 1992 firms'; hence, output does not rise much in the *Base*.

The interest rate drops 0.3%p except for *Worker*. Based on the estimates in Holston et al.'s (2017), real interest in the U.S. decreases 2.4% points between 1992 and 2013. The changes between the 1992 and 2013 steady states account for 15% of the decline, suggesting that the changes in the earnings risks could be a reason for the declining real interest rate.[28] Lastly, it is worth noting that unlike earnings distribution or wealth distribution, the changes in the firm productivity process and the matching between workers and firms have larger effects on capital and output.

5.4. Discussion

The counterfactual exercises reveal that the changes in the labor productivity process are the main drivers in the rise in earnings inequal-

[28] Studies that focus on the decline of the real interest rate suggest that low GDP growth rate (Holston et al. (2017)) and aging population (Carvalho, Ferrero, Nechio, et al. (2017)) can be reasons for the decline.

ity. In various inequality measures – variance of earnings, earnings share by quintile and top 10%, earnings Gini index, and 90/10 ratio – the changes in the ε process make the largest differences. The changes in worker-firm matching drive the rise in firm inequality as well as the rise in segregation and sorting. However, they do not increase earnings inequality but mitigate it. The changes in the firm productivity process mitigate the rises in earnings inequality too.

It is beyound the scope of this paper to investigate the causes of the changes that are imposed in the counterfactual exercises, but the natural question would be; what are the causes of those changes? There are many reasons that can be behind these shifts. I decompose earnings into worker factor and firm factor and interpret those as productivities. But if one drops the competitive factor market assumption, then the changes in the productivities can be further decomposed.

One can decompose ε further into an efficiency of labor and a price of the unit and study each element's trend and why it has changed over time. In fact, most of the literature on earnings inequality tried to understand the changes in skill premium. (Acemoglu (2002), Katz and Murphy (1992), Goldin and Katz (1998), Autor et al. (1998), Krusell et al. (2000)), and Card and Lemieux (2001)) Heathcote et al. (2010) follow this approach and Alon (2018) shows that job specialization leads to human capital investments on a narrower sets of skills, and that a lack of such investment causes the rise in skill premium.

The firm productivity process is calibrated to match the wage rate differences among firms. Consistent with the recent evidence by Bloom et al. (2018) that show the declining large firms' wage premium, the firm productivity distribution is less dispersed in 2013 compared to 1992 in the calibration. However, without the competitive factor market assumption, the changes in firms' compensation differences can be decomposed into the changes in firms' measured productivity distribution and the changes in the pass-through of productivity to wages.

Using data, productivity can be measured as a residual of production function[29] – a fraction of output that is not explained by inputs – and it is known that there is large dispersion in measured productivity across establishments within narrowly defined industries (Foster, Grim, Haltiwanger, and Wolf (2017)). Decker, Haltiwanger, Jarmin, and Miranda (2018) and Kehrig (2015) show that the measured productivity dispersion has been increasing in the manufacturing industry. Berlingieri, Blanchenay, and Criscuolo (2017) document an increase in productivity dispersions for both the manufacturing and services industries. The declining large firm wage premium could have been caused by the changes in pass-through of profit to wages. There is suggestive evidence that supports this hypothesis. For example, Kehrig and Vincent (2017) and Autor et al. (2017) show that there have been reallocations of production toward super-productive/large firms and a downward trend of the labor

[29] See Syverson (2011) for a definition and measurement of productivity.

share of those firms over time. This implies that these large firms may have decreased profit pass-through. [30]

What determines degrees of profit pass-through by firms? Hirsch and Müller (2018) and Farber, Herbst, Kuziemko, and Naidu (2018) focus on effects of institutions on rent sharing between firms and workers. Hirsch and Müller (2018) show that unionized firms having larger wage premiums using German data, and Farber et al. (2018) show that union households' income premiums over the past eight decades and declining union membership increase income inequality. Instead of institutions, Webber (2015) and Gouin-Bonenfant (2018) focus on the impact of imperfect competition. Webber (2015) estimates labor supply elasticity using LEHD data and shows the effect of monopsony power on earnings distribution. In Gouin-Bonenfant's (2018) model, productive firms poach workers and offer higher wages in order to increase their market share. His model predicts that an increase in the productivity differential among firms effectively shields high productivity firms from wage competition, which implies that the wage premium of productive firms will decline as a firm's productivity distribution becomes more dispersed.

In the calibration, the separation rate and the number of searchers fall to be consistent with the declining job flows. Among others, Molloy, Trezzi, Smith, and Wozniak (2016) and Hyatt and Spletzer (2013)

[30] If this is closer to the truth, then output increases due to the rise in sorting and segregation could have been much higher.

document the trends and explain the reasons behind the decline. At the same time, the job transition probability difference between high-skilled and low-skilled workers becomes larger in 2013 to match the rise in segregation and sorting. Outsourcing could be the reason that reflects this observation. Presumably, the development of information technologies has increased the efficiency of search in the labor market, resulting in the rise in sorting and segregation.

5.5. Caution

The results should be interpreted carefully. The variance decomposition results in Song et al. (2015) are important to pin down the dispersion of workers and firm distribution. However, Song et al.'s (2015) results are computed using the sample that drops firms that hire less than 20 workers – 88% of firms and 18% to 20% workers, which is not a small amount.[31]

Also, various aspects of the labor market have not taken into account in the counterfactual exercise. While the model assumes same separation rate, unemployment rate, and unemployment duration to all workers, there is much evidence that the overall labor market condition has become tougher for low-skilled workers. Wolcott (2018) documents the increasing employment gap between low-skilled and high-skilled work-

[31] The share of firms and workers that are dropped are based on BDS. Since Song et al.'s (2015) definition of firm is EIN, these shares might not be accurate.

ers and shows that a shift in demand away from low-skilled workers is the leading cause of the increasing gap. Charles, Hurst, and Schwartz (2018) report the large decline in employment rates of low-skilled workers in the manufacturing sector. Cairo and Cajner (2018) show more educated workers experience lower unemployment rates and lower employment volatility. These heterogeneous labor market conditions to different workers could have affected the changes in earnings distribution as well as wealth distribution and aggregate variables.

6. Conclusion

Motivated by empirical evidence that shows rising inequality in individuals' earnings and firms' employment and revenue, I propose a model with two-sided heterogeneity and the labor market friction to study these trends. I have calibrated the model to match the earnings distribution, individual and firm components in the variance of earnings, and the firm distribution. The model also matches the moments in earnings dynamics and the wealth distribution. From the counterfactual exercises, I find that the changes in the labor productivity process explain most of the rise in earnings inequality and the majority of the firm concentration is driven by changes in the firm productivity distribution and the worker allocation over firms. Surprisingly, the shifts in firm productivity distributions and worker-firm matching mitigate the rise in earnings inequality.

The results require several caveats. Some calibration targets are computed from the sample that dropped small firms. Although I mapped the available data to the model accordingly, future work could try to investigate the changes in small firms more closely. The firm productivity distribution is pinned down to target compensation differences among firms; therefore, it may not be suitable to study the implications to aggregate variables such as output.

There are several avenues related to this work that deserve further study. Studying causes of the changes in the stochastic processes and their effects on earnings inequality will further deepen our understanding of the rise in earnings inequality. As discussed in the paper, it could well be that large firms have monopsony power and exert it. Separating firms' compensation decision from productivity would be necessary to investigate this further. A model that can incorporates a more flexible wage rate to different workers within a firm and micro data that can discipline the model would be necessary. Estimating the earnings process allowing for two-sided worker-firm unobserved heterogeneity in a rich environment such as Bonhomme, Lamadon, and Manresa (2015) would allow us to take a model with richer wage settings to the data.

The interaction between investment and inequality needs further study. In this framework, firms' capital demand is a function of their productivity and an interest rate given the frictionless capital market assumption. It allowed me to decompose the factors affecting the inequality neatly but misses the interaction between firms' investment decisions and their effect on the inequalities. For example, if firms have to choose the capital in advance, it may lessen the effect of the productivity shock. Assuming persistent productivity, a firm with high productivity today will choose a high level of capital for the future. If the firm is hit by a shock, having large capital may prevent the firm from experiencing a large drop of output and a wage cut. At the same time it may interfere with small firms' growth. Given the highly concentrated firm distribution, a capital friction may affect earnings distribution as well as the aggregate variables.

Appendix A. Numerical solution method

Thanks to the search option, the value function is not concave nor smooth. I use the generalized endogenous grid method by Fella (2014), which extends Carroll's (2006) method for non-smooth and non-concave problems. Carroll's (2006) method uses the first order condition to avoid root finding operations. It is fast and accurate, but to use the method a value function has to be concave, so the first order condition is necessary and sufficient for an interior optimal choice. Fella's (2014) method relies on the fact that even in a non-concave region of the value function, the first order condition is still necessary for an interior local maximum. Intuitively, the value function points inwards at kink, and a kink point can never be an optimal choice. Therefore, points in the non-concave region are still candidates for interior global maximum.

Aside from searching, households' problem is choosing a' given (ε, a, z). In brief, the algorithm identifies the non-concave region and checks whether the $(a', a|\varepsilon, z)$ pair in the region is optimal or not using grid search. If a pair is indeed optimal the algorithm keeps the pair; if not, the pair is dropped. Outside of the non-concave region all pairs are kept.

Since some pairs are dropped in the non-concave region and selected pairs may vary in each iteration, convergence of the value function is sensitive to the grid size. In general the grid needs to be fine enough, which increases computing time. To avoid setting fine grid from the beginning, I set a relatively sparse grid at first but refine the grid while updating the value function.[32] I execute the refinement in following way:

1. In $i+1$th iteration, if $max|W^i - W^{i+1}|$ stops decreasing, pause the iteration and find the point where $max|W^i - W^{i+1}|$ jumps
2. Around the point, add a few points
    - The set of points selected by the algorithm will be larger and stable going forward
3. Start a new iteration with the extended grid

In my model, given (ε, z) there is a threshold asset level with respect to searching; below it, certain a workers do not search. Since these thresholds vary with (ε, z), it is hard to know where the kink points will be. This extension allow me to solve the model efficiently while having fine grids where they need to be.

[32] The insight of this refinement is mentioned in Fella (2014), but it was not implemented.

Appendix B. Additional figures and tables

Figure 7. Labor productivity

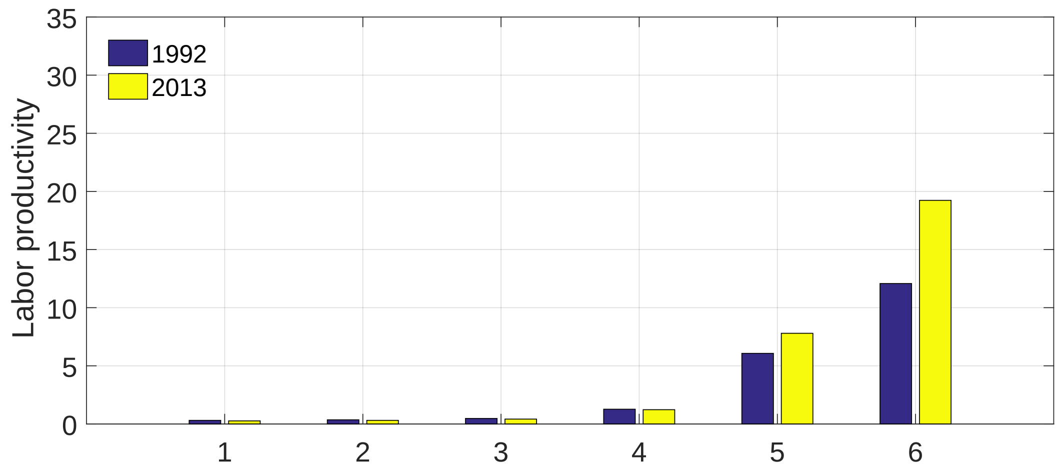


Figure 8. Wage rate

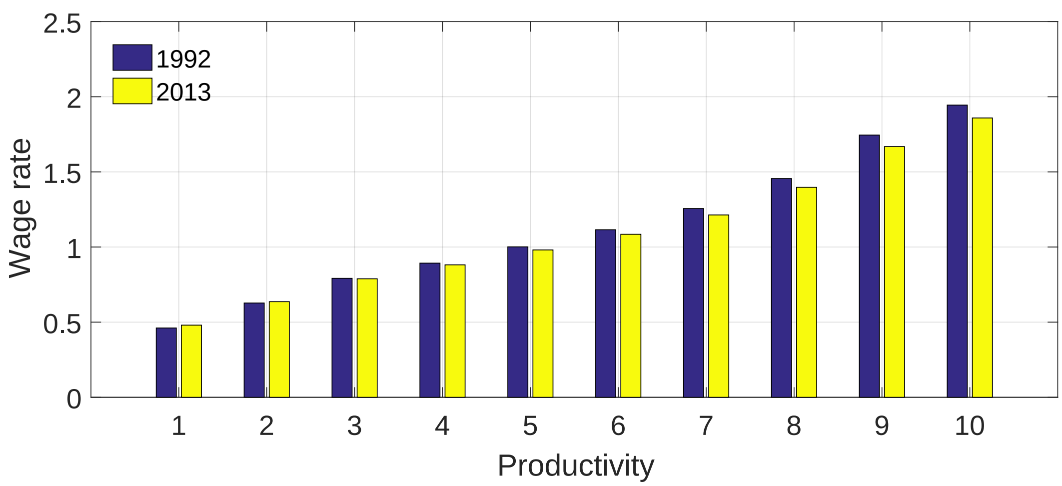
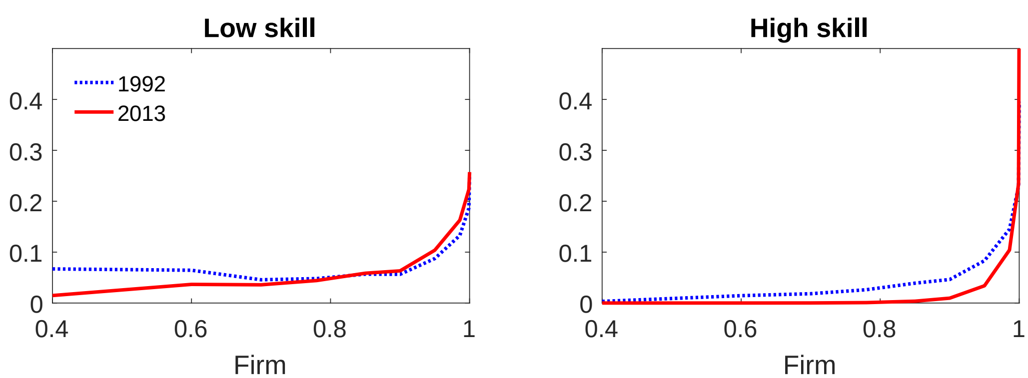


Figure 9. Job transition probability



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