

# Firm age, business cycles and aggregate labor market dynamics

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November 8, 2011

## Abstract

Job creation and destruction rates fall with a firm's age, young businesses have higher exit rates, conditional on survival they grow faster and given their employment shares they create relatively more jobs than older firms. In fact, recent studies show that the well established negative relationship between a firm's size and its growth rate vanishes once its age is taken into account. I extend these findings by showing that, compared to old firms, employment growth in young firms is more volatile and that business start-ups are important for unemployment rate developments. Next, I build a general equilibrium model with heterogeneous firms that is consistent with these cross-sectional facts and delivers realistic aggregate labor market dynamics. The model is used to evaluate a government policy supporting young firms, a measure proposed under the recent "Startup America" initiative of the White House. The results suggest that such a policy should focus mainly on reducing barriers to entry. Supporting existing firms disrupts the selection process of successful firms, reduces average firm productivity, and results in lower levels of output.

*Keywords:* firm age, firm growth, unemployment, heterogeneity

*JEL Classification:* E24, E32, J64

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\*I am grateful to my advisor Wouter den Haan, Bart Hobijn, Matija Lozej, Vincent Sterk, Robert Swift and Tim Willems for their suggestions and comments. Address: University of Amsterdam, Valckenierstraat 65-67, 1018XE Amsterdam, The Netherlands. Email: [p.sedlacek@uva.nl](mailto:p.sedlacek@uva.nl)

# 1 Introduction

The most recent economic downturn in the U.S. was accompanied by a severe deterioration of the labor market. The effects of the economic downturn, however, were not the same across different firms. Employment in both young and small firms fell much more than in older, larger businesses.<sup>1</sup> There is a long list of studies focusing on the role of firm size for the growth of businesses and the importance of *small* firms for aggregate job creation. Recently, however, a few studies have pointed out that it is *young* firms that are important for aggregate job creation and that young firms tend to grow faster than older ones. Moreover, Haltiwanger, Jarmin, and Miranda (2010) suggest that there is no link between a firm's size and its growth, once its age is taken into account. This poses new challenges for economic research as it questions current theoretical models of firm growth that rely on the assumption of decreasing returns to scale in production. Such an assumption automatically creates a negative relationship between a firm's growth and its size, independent of its age.

In this paper I examine, both empirically and theoretically, the freshly highlighted link between a firm's growth and its age and the impact young firms have on the aggregate labor market. The first contribution of this paper is to extend current empirical findings by examining the employment behavior of firms of different age and the role young firms play in shaping aggregate labor market dynamics over the business cycle. Next, while there are many theoretical models of firm growth, in my understanding of the literature, none of them explicitly focuses on firm age as an important factor for business growth.<sup>2</sup> Thus, the second, main contribution of this paper is a novel general equilibrium model with *heterogeneous* firms in which a firm's age is the main driver of its growth and which captures the cross-sectional facts related to firm age. Given the stubbornly high unemployment rates in many countries affected by the recent crisis, policy questions related to job creation have gained in importance. Thus, the presented model is used as a laboratory in which I evaluate the effects of a government policy aimed at supporting young firms. Such a measure is present for instance in the recent "Startup America" initiative of the White House.

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<sup>1</sup>Jobs declined by 10.4% in firms with fewer than 50 employees, while larger businesses reported a 7.5% drop (Sahin, Kitao, Cororaton, and Laiu, 2011). Similarly, firms 5 years of age and younger experienced an employment fall of 14.8%, while jobs in older firms were reduced by 2.8%.

<sup>2</sup>See for example Jovanovic (1982), Hopenhayn and Rogerson (1993), Acemoglu and Hawkins (2010), Elsby and Michaels (2010), Kaas and Kircher (2011), Moscarini and Postel-Vinay (2010b).

Recent empirical studies that have devoted their attention to analyzing the link between firm growth and its age have shown that young firms have higher exit rates but conditional on survival they grow faster than older businesses. Furthermore, job creation and destruction rates fall with firm age and, given their employment shares, young businesses create relatively more net jobs compared to old firms. Finally, young firms are also found to be mainly small, but small firms are not necessarily young.<sup>3</sup>

I confirm these findings over a larger sample, including the most recent recession, by analyzing the latest version of the Business Dynamics Statistics (BDS) database. I then extend these facts by documenting two new empirical regularities. First, compared to old firms, employment growth in young businesses is more volatile contributing to unemployment increases during and right after recessions and boosting employment growth later on in expansions. Second, business start-ups are important for developments of the unemployment rate. This is especially evident during the most recent recession when lower than average firm entry alone accounted for almost 1/5 of the observed unemployment rate increase.

The main contribution of this paper is a novel general equilibrium model with search and matching frictions and heterogeneous firms which captures the above-mentioned cross-sectional facts and with which one can examine policy questions related to firm age. The model allows for rich firm dynamics in which businesses of different age have endogenously different employment behavior. Based on the expected benefits of operating a firm, new firms endogenously enter the economy obtaining an initial firm-specific productivity level (business idea). These individual productivity levels then persistently evolve over time. Based on aggregate and individual business conditions, active firms produce and decide on (costly) hiring of unemployed workers. If firm-specific conditions are so bad that it no longer pays off to remain in operation, firms shut down. In addition, firms can shut down and workers can be dismissed for exogenous reasons.

In contrast to common practice in the literature, firm growth in this model does *not* rest on the assumption of decreasing returns to scale in production. Firms in this model operate a production technology that uses labor as its only input and that features *constant* returns to scale.<sup>4</sup> Assuming decreasing returns to labor makes small firms grow

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<sup>3</sup>See for instance Haltiwanger, Jarmin, and Miranda (2010), Neumark, Wall, and Zhang (2006), Ibsen and Westergaard-Nielsen (2011), Halabisky (2006), Ayyagari, Demircuc-Kunt, and Maksimovic (2011).

<sup>4</sup>Many labor market models abstracting from capital also use constant returns to scale production technologies. See for example Moscarini and Postel-Vinay (2010b) or the classic Mortensen and Pissarides (1994) paper and many others that build on it.

faster *automatically* (irrespective of their age), because they can take advantage of higher marginal products of labor as they are further away from their optimal size. This directly contradicts the findings of Haltiwanger, Jarmin, and Miranda (2010) that within a given age group there is no systematic link between firm size and firm growth. The setup in this paper does not rely on decreasing returns to scale and implies that a small and a large firm with the same level of productivity will advertise the *same* number of jobs. A constant returns to labor technology, however, also means that the concept of optimal firm size vanishes. Nevertheless, the presence of exogenous worker dismissals and exogenous firm exits results in a well-defined firm size distribution.

The costs related to hiring new workers are assumed to be convex. Assumptions of increasingly costly factor adjustment are common in many models.<sup>5</sup> In the context of this paper such an assumption implies that large, rapid, changes in employment come with increasingly high costs. In other words, firms that are productive enough to expand do so in a gradual manner.

To solve the model, I employ a standard solution technique for heterogeneous agent models proposed in Krusell and Smith (1998). However, the properties of the model are such that one quickly runs into the curse of dimensionality. For this reason, I propose to solve a *modified* maximization problem which does not have this unfavorable property. Accuracy checks show that this is a valid procedure for the model at hand.

The calibrated model is consistent with the established empirical findings concerning firm age characteristics. The key to understanding the performance of the model is the inherent selection mechanism of successful firms. Business start-ups are characterized by a wide range of business ideas (productivity levels). Relatively less productive firms shut down early in their lives leaving only the more successful businesses to expand and grow old. In other words, the risk of going out of business as well as job destruction rates are higher for younger firms. Moreover, conditional on survival, young firms tend to grow faster than older businesses. The reason behind this is twofold. First, some business start-ups initially have high productivity levels, but as they grow old, they revert back to the (common) mean of firm-specific productivity. Second, given a productivity level a small and a large firm will hire the same number of workers. However, smaller firms exogenously loose less workers (in absolute terms) leaving them with relatively more resources for expansion compared to larger businesses. Therefore, given that young firms

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<sup>5</sup>As for example in Acemoglu and Hawkins (2010), Bloom (2009) Kaas and Kircher (2011), Merz and Yashiv (2007).

are mainly small, the combination of these two effects means that young businesses create relatively more (net) jobs, job creation rates fall with a firm's age and employment growth in young firms is more volatile than in older ones.

The model is not only consistent with firm-level dynamics, but it also generates realistic business cycle statistics of aggregate labor market variables, both in terms of co-movement and volatility. Moreover, firm heterogeneity together with the endogenous process of firm entry, growth and exit, create two new propagation mechanisms that are not present in a standard model assuming a representative firm. Their interaction generates greater persistence and richer dynamics than those predicted by a standard model.

The framework in this paper stresses the importance of firm heterogeneity, especially the dimension of firm age, providing a natural setting in which to analyze policy measures aimed at supporting young firms. Such measures have been proposed for instance under the recent "Startup America" initiative of the White House. In the presented model there is a role for government intervention as the model calibration implies that firms get only a small fraction of output. This results in too few business start-ups and too much firm exit. The fact that the majority of firm selection happens early in a firm's life (young firms have higher exit rates) justifies the consideration of a subsidy aimed at young firms.

The results imply that subsidizing firm entry increases welfare as more business start-ups help reduce unemployment and increase output. However, if the government focuses its resources only on existing young firms aggregate output actually falls. The reason is that a subsidy for existing firms enables relatively less productive firms to remain in business, crowding out entrants. As a result, average firm productivity declines and the re-allocation of workers from relatively less productive firms to more efficient businesses is hampered. Thus, the model suggests that policy measures should focus on reducing barriers to entry and thereafter they should quickly be withdrawn so that the economy can pick its own winners.

The paper is structured as follows. In Section 2, I provide a short overview of studies related to this paper. Section 3 establishes empirical facts about firm age characteristics in the U.S. economy. Section 4 then builds a general equilibrium model aimed at explaining the established empirical facts. Section 5 describes the solution method and Section 6 provides the calibration of the model. In Section 7, I summarize the model predictions for firm age characteristics and aggregate labor market dynamics. Section 8 uses the model to analyze the impacts of a government subsidy supporting young firms and finally, Section 9 gives some concluding comments.

## 2 Related research

In this section, I provide a short overview of three areas of research that are related to this paper. The topics include the relationship between firm size and firm growth (while *not* accounting for firm age), the link between firm age and growth of businesses and finally, theoretical models incorporating firm heterogeneity.

The contributions of Birch (1981) and Birch (1987) sparked interest in the link between firm size and firm growth. The central message of Birch’s research was that small firms are the most important source of job creation in the U.S. economy, since they tend to grow faster than older businesses. Davis, Haltiwanger, and Schuh (1996) criticized the analysis on the basis of it being subject to the “regression fallacy” or “regression to mean effect”.<sup>6</sup> Instead, the authors propose a different firm size definition to weaken this statistical pitfall. Using manufacturing plant data, they then conclude that the regression fallacy fully accounts for the negative relationship between size and growth. On the other hand, using the National Establishment Time Series (NETS) database covering the entire U.S. economy and avoiding the above caveats, Neumark, Wall, and Zhang (2011) document an inverse relationship between net growth rates and firm size.

Similar studies have also been conducted on data from other countries. Davidsson, Lindmark, and Olofsson (1998) use Swedish data and conclude that small firms contribute more to net job creation than large ones. Baldwin and Picot (1995), Barnes and Haskel (2002), Broersma and Gautier (1997), and Voulgaris, Papadogonas, and Agiomirgianakis (2005) come to similar conclusions using data from Canada, the United Kingdom, The Netherlands and Greece, respectively.

While none of the studies above took into account the effect of firm age, they were a starting point for more recent research that did draw its attention to age as the driver of firm growth. Using the BDS database, Haltiwanger, Jarmin, and Miranda (2000) document the high job creation and destruction rates of young businesses in the period between 1992 and 2005 in the U.S. economy. Neumark, Wall, and Zhang (2006) find an important role of firm start-ups in the high job creation share of small firms in the NETS database. Moreover, Haltiwanger, Jarmin, and Miranda (2010) argue that once

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<sup>6</sup>When using a given base year to classify firms into size categories, two types of errors can occur. Businesses that are not small can be earmarked as small either because of measurement error or a transitory negative shock to their employment levels. In both cases the firm will tend to “grow faster” in the following periods as their employment levels revert back to their mean. The opposite logic can be used for large firms leading to a downward bias in the relationship between firm size and growth.

one controls for firm age, the negative relationship between a firm's size and its growth vanishes. A similar conclusion is made in Ibsen and Westergaard-Nielsen (2011) for Danish data. Halabisky (2006) shows that in Canada young firms account for the bulk of net job creation. Using a cross-section of 99 countries, Ayyagari, Demircuc-Kunt, and Maksimovic (2011) conclude that small and young firms have higher job creation and destruction rates.

The theoretical model of firm growth, entry and exit, presented in this paper, is closely related to several papers, starting with the early study of Jovanovic (1982). In his model, new firms grow faster and are more likely to fail than older ones, as they learn about their efficiency level. Hopenhayn and Rogerson (1993) present a general equilibrium model including rich firm dynamics (without aggregate uncertainty) and analyze the welfare impacts of firing taxes. More recently, Acemoglu and Hawkins (2010), Elsby and Michaels (2010), Kaas and Kircher (2011), Moscarini and Postel-Vinay (2010b) and Schaal (2010) extend the Mortensen-Pissarides model to include multiworker firms. While Acemoglu and Hawkins focus on the implications of the firm size distribution for unemployment and vacancy persistence, Elsby and Michaels show that a model with endogenous separations and a role for firm size can account for the business cycle features of aggregate labor market variables. Kaas and Kircher address the question of efficiency in search and matching models with multiworker firms and Moscarini and Postel-Vinay provide a theoretical underpinning for their empirical finding that larger employers fluctuate more than smaller ones. Finally, Schaal uses a rich heterogeneous firm model to analyze the interaction of aggregate productivity shocks and uncertainty shocks to explain the simultaneous and persistent increase in unemployment and a sharp rise in labor productivity observed in the recent recession.

### **3 Firm age and job creation in the U.S. economy**

This section uses the most recent version of the Business Dynamics Statistics database to confirm the findings of previous studies related to average firm age characteristics across business cycles over a larger sample which includes the most recent downturn. Next, I extend these findings and document two new facts concerning young firms and their business cycle behavior.

The BDS is a publicly available database constructed by the Census Bureau. It covers approximately 98% of U.S. private employment and contains information on employment stocks and flows. The data is annual and runs from 1977 to 2009 and is broken down by

location, industry, firm size and firm age.

I report the empirical findings in two blocks. First, facts regarding averages over the sample period between 1992 and 2009. This is a compromise between analyzing the data over a longer sample and including a rich enough age structure. Second, empirical findings related to business cycles. For this second block, I choose to differentiate only between firms younger than six years and the rest. This enables me to extend the sample period such that it starts in 1982. While the first block is an extension to previous studies based on an earlier version of the BDS database that ended in 2005, the second block of empirical facts is to my knowledge new. Before analyzing the data, I define the concepts of job creation, job destruction, firm size as well as the age categories that will be used throughout the paper.

### 3.1 Definitions

I define job creation and destruction, firm size following Davis, Haltiwanger, and Schuh (1996) and I specify three firm age groups

- **gross job creation** as the sum of employment gains over all businesses whose employment level has increased during the last year,
- **gross job destruction** as the sum of employment losses over all businesses whose employment level has decreased during the last year,
- **net job creation** as the difference between gross job creation and gross job destruction,
- **firm size** is the simple average of firm employment in year  $t$  and  $t - 1$ ,<sup>7</sup>
- **new firms** are those younger than 1 year,
- **young firms** are those 5 years of age and younger (including new firms),
- **old firms** are those older than 5 years.

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<sup>7</sup>This firm size definition is also known as the “current” or “average” firm size. An alternative, perhaps more natural firm size concept is simply firm employment in period  $t$  (“base year” firm size definition), Davis, Haltiwanger, and Schuh (1996) introduced the “current” firm size definition to diminish the “regression fallacy” as mentioned in the literature overview.

## **3.2 Average firm age characteristics across business cycles**

In this section, I describe the general characteristics of the firm age distribution, the related firm dynamics and facts about job creation and job destruction according to firm age.

### **3.2.1 The distribution of firms**

On average 40% of all firms are younger than 6 years. New firms, a subset of young businesses, account for about 10% of all firms. Firms in the oldest category (16 years and older) account for almost 30% of all businesses.

The BDS database also reports statistics according to the joint breakdown into firm age and size. Figure 1 shows the firm age shares in a given size category, while Figure 2 depicts the opposite ordering and shows firm size shares in a given age category. Figure 2 documents that in all age categories the size shares fall monotonically as firm size increases. On the other hand, this is not apparent in Figure 1 where the age shares for small firms do not display a decline as firm age increases. Hence, it is the case that young firms are mostly small (Figure 2), but small firms are not necessarily young (Figure 1).

### **3.2.2 Size, age and firm growth**

The U.S. economy is highly dynamic with on average 10% of all firms shutting down and 10% of all firms being start-ups each year. Table 1 documents that younger firms have higher exit rates than older ones, with young firms accounting for almost 1/2 of all firm closures.

Not only do younger firms have higher exit rates but, conditional on survival, they also tend to grow faster than more mature firms resulting in a strong “up-or-out” tendency. Haltiwanger, Jarmin, and Miranda (2010) document the negative relationship between a firm’s age and its growth. Using a non-parametric specification and controlling for detailed industry and year fixed effects, they regress net employment growth rates on firm age classes and find that young firms grow faster than older ones with new firms being crucial for this result. Furthermore, they also investigate the case when size is added as an additional explanatory variable. In this case, they no longer find that small firms have systematically larger net employment growth rates than larger businesses, as previously documented in the literature.

### 3.2.3 Job creation and destruction according to firm age

More than 1.5 million net new jobs are created each year on average. This number, however, hides a large amount of churning, since annually 17.5 million jobs are created and 16 million jobs are destroyed on average. Put differently, almost 30% of all jobs are either destroyed or newly created.

**Rates.** Table 2 shows the job creation and destruction *rates* according to firm age.<sup>8</sup> Both job creation and destruction rates drop gradually with firm age. The stunning feature of Table 2 is that only new firms have positive net job creation. However, gross job creation is large in all age categories and thus one should not conclude that it is only new firms that account for all job creation in the economy.

**Shares.** The above suggests an important role of young businesses in job creation. However, young firms are mostly small and thus it is not clear whether high job creation rates also translate into a large fraction of newly created jobs. Table 3 shows the shares of gross job creation and destruction of given firm age groups in the total. The last column reports the respective employment shares. The clear “outlier” is the group of new firms that accounts for 17.4% of all jobs created in expanding firms, while their employment share is only 1.4%.<sup>9</sup> Young firms create 34.3% of all jobs in expanding businesses and they destroy 21.5% of all jobs in contracting firms, even though they account for only 15.1% of total employment. Thus, young firms are important for aggregate job creation as they create a disproportionately large amount of jobs.<sup>10</sup>

## 3.3 Firm age characteristics over the business cycle

The previous paragraphs spoke about the average importance of young firms for job creation across business cycles. In this section, I first examine the cyclical properties of young firms’ employment growth rates. Second, I conduct two counter-factual scenarios to highlight the importance of young firms for aggregate labor market dynamics.

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<sup>8</sup>The 200% job creation rate of new firms is an artifact of the firm size definition. Job creation and destruction rates are calculated as the total number employment gains (losses) over all businesses whose employment level has increased (decreased) during the last year divided by firm size. The firm size definition is based on the simple average of period  $t$  and  $t - 1$  employment levels. In the case of new firms this results in firm size being half of period  $t$  employment, since these firms did not exist in period  $t - 1$ .

<sup>9</sup>Using the base year size definition the employment share would be twice as large, 2.8%.

<sup>10</sup>One should note that while the number of young firms is relatively stable over time, the total number of firms is growing. However, as indicated in the paragraphs above, it is important to put the job creation and destruction shares of young firms in relation to the respective employment shares, which were also declining over the given sample. Therefore, the overall picture that young firms create a disproportionately large amount of net jobs still holds.

### 3.3.1 Employment growth rates of young and old firms

Employment growth rates of both young and old firms are procyclical, with that of young businesses being 2.5 times more volatile than that of old firms. The correlation coefficients between the cyclical component of the unemployment rate and the two employment growth rates are  $-0.57$  and  $-0.63$  for young and old firms, respectively.<sup>11</sup>

To further understand this cyclical pattern, one can look at the business cycle properties of the *difference* between employment growth rates of young and old firms. This was proposed by Moscarini and Postel-Vinay (2010a) for the group of small and large firms. Figure 3 plots this differential growth rate together with the unemployment rate.<sup>12</sup> Young firms contract faster or expand slower than older firms (a low differential growth rate) during recessions and early in recoveries and they grow faster or contract slower than older ones (a high differential growth rate) later on in expansions. The correlation coefficient of the two series is  $-0.62$ .<sup>13</sup>

### 3.3.2 Understanding the importance of young firms

To further analyze the impact young firms have on aggregate labor market outcomes, I consider two counterfactual scenarios. First, what would the unemployment rate look like if young firms' job creation and destruction rates were the same as those of old firms? Second, I focus on the latest recession which was especially hard on young firms. I ask what would the aggregate unemployment rate be if young firms' job creation and destruction behaved as it did on average during the previous recessions?

**Job creation and destruction of young businesses as that of old firms.** I construct a counterfactual aggregate employment level by replacing actual job creation and destruction rates of young firms by those of old businesses. The difference between actual employment and this counterfactual value is then added to the unemployment rate. The counterfactual unemployment rate is about 2 percentage points higher on average, as young firms' net job creation rates are higher than those of old businesses.<sup>14</sup>

More interestingly, there are also some differences in dynamics. Figure 4 shows the

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<sup>11</sup>The cyclical components are extracted in this paper using the HP filter with a smoothing coefficient of 6.23. This value (for annual data) is based on the recommendation in Ravn and Uhlig (1997).

<sup>12</sup>Both unemployment and the differential growth rate are detrended with a quadratic time trend.

<sup>13</sup>Detrending with an HP filter instead delivers a slightly lower correlation coefficient of  $-0.43$ .

<sup>14</sup>Note that this level effect is likely to be underestimated as I leave the employment of old firms unchanged. Specifically, I do not account for the effect that lower net job creation rates of young firms translate into lower employment levels of old firms in the future.

difference between the actual and mean-adjusted unemployment rates together with its decomposition into the contributions of firm entry, exit, job creation and destruction of existing firms.<sup>15</sup> A positive difference between the two unemployment rates means that the different dynamics of young firms' job creation and destruction compared to those of old firms contributed to unemployment increases. Similarly, a negative difference indicates that young firms' dynamics, relative to those of old firms, contributed to a decrease in the unemployment rate.

The figure shows that the different dynamics can account for up to 0.9 percentage points of unemployment, which is about 1.5 times the standard deviation of unemployment in the given sample. The decomposition shows that firm entry accounts for the largest share in the observed difference between actual and counterfactual unemployment.<sup>16</sup>

#### **Job creation and destruction of young firms as in an average recession.**

I now zoom in on the latest recession only. I ask what would the unemployment rate look like if young firms behaved as they did on average during the other recessions in the sample? To answer this, I detrend (log) real GDP, job creation and destruction with a linear time trend and calculate the average response of young firms' job creation and destruction to a 1 percentage point decrease in real GDP over the previous recessions.<sup>17</sup> I then use this average response and the observed drop in real GDP to create counterfactual (un)employment in the most recent downturn. Based on the average response from previous recessions, employment in new firms would have fallen by about 5% in the latest recession, while it actually dropped by almost 30%.<sup>18</sup>

Table 4 shows the actual and counterfactual unemployment rate for the latest recession. The difference in 2009 is more than 1 percentage point. Thus, only the fact that young firms were hit especially hard during the latest downturn accounts for 25% of the unemployment run-up during the latest recession.

One can decompose this difference into contributions of entry, exit, job creation and destruction of continuers. The lion's share of the unemployment differential is because the level of start-ups is lower than it usually is in recessions. The second most important

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<sup>15</sup>The mean-adjusted counterfactual unemployment rate is constructed by replacing young firms' job creation and destruction rates by those of old businesses, but keeping the average levels unchanged. Simply subtracting the average unemployment rate delivers similar results.

<sup>16</sup>Note that old firms have zero entry rates by definition. The difference thus arises because business start-ups are above or below their average value.

<sup>17</sup>Using a quadratic time trend instead changes little.

<sup>18</sup>The average decline of 5% may seem small. However, employment in new firms typically continues in its decline early in the recovery phase. The most recent crisis is therefore unique in the sense that employment in new firms plummeted already during the recession.

contributor was lower job creation of continuing firms. Higher job destruction (either due to firm exits or firing in continuing firms) contributes little.<sup>19</sup> Thus, the underlying message of these counterfactual scenarios is that young firms, and especially entrants, are important for aggregate labor market dynamics.

### 3.4 Summary of empirical facts

- **Importance of age for firm growth:** size does not matter for firm growth once age is taken into account (Haltiwanger, Jarmin, and Miranda, 2010). Young firms have higher exit rates and conditional on survival tend to grow faster than older businesses.
- **Firm age/size distribution:** 40% of firms are young and they are mainly small, while small firms are not always young.
- **Job creation and destruction across business cycles:** job creation and destruction rates fall with firm age. Young firms create a disproportionately large number of (net) jobs compared to their employment shares.
- **Cyclicity of young and old firms:** employment growth rates of both young and old firms are procyclical. Young firms are more volatile than older businesses contributing to unemployment increases during and right after recessions and boosting employment growth later on in expansions.
- **Importance of entry:** firm entry plays a crucial role in the effect of young firms on aggregate labor market dynamics accounting for large shares of total employment declines during recessions, especially in the latest downturn.

## 4 Model

I turn now to building a general equilibrium model aimed at capturing the above empirical facts. The economy is populated by a large number of heterogeneous firms that differ in their productivity levels which evolve (persistently) over time. Each firm operates a production technology that uses labor as its only input. Firms obtain labor by hiring workers on a frictional labor market.

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<sup>19</sup>In 2008 job destruction in continuing firms was even lower than the historical average in the previous recessions.

In each period, based on aggregate and firm-specific conditions, existing firms decide whether or not to stay in the economy. At the same time, new firms endogenously enter the market upon which they obtain an initial productivity draw from a wide range of “business ideas”. All firms that remain active in the economy are subject to aggregate and firm-specific productivity shocks. Based on their development, they produce and decide on whether to expand or shrink their workforce. Hence, even when aggregate productivity is fixed, the model still generates rich firm dynamics. Some firms start up and some firms that shut down, some businesses will expand, while others will contract. Next, I turn to explain the model in more detail.

#### 4.1 Timing

The timing of events in this economy is depicted in Figure 5. At the beginning of the period, *before* any shocks are revealed, new firms enter the market. At the same time, incumbent firms choose whether to continue production or shut down. After entry and exit decisions have been made, the innovations to both the aggregate and idiosyncratic levels are revealed.<sup>20</sup> Active firms pay an operational cost, produce output and pay their workers. They also decide how many vacancies to post and pay the appropriate cost. At the end of the period, a fraction  $\delta$  of all firms exogenously shuts down and all their employees enter the unemployment pool. In addition, a fraction  $\rho_x$  of workers in employment relationships with existing firms exogenously lose their jobs. All workers in the unemployment pool, including those entering at the end of the period, are ready to find a job in the next period.

#### 4.2 Household behavior

Households are assumed to be risk neutral.<sup>21</sup> The household consists of a continuum of ex-ante *homogeneous* workers of unit mass. The members of the household pool their incomes from firm ownership, employment and non-employment activities and spend it on consumption. The model abstracts from any investment or labor force participation decisions. The household thus maximizes expected life-time utility subject to the following

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<sup>20</sup>Assuming that entry and exit occurs prior to observing the shocks follows Hopenhayn and Rogerson (1993). Moreover, this setup ensures that both the exit decision and the vacancy posting decisions are based on the expected continuation value of the firm. Hence, it will never happen that a firm chooses to post vacancies, incurs the associated costs, and then chooses to exit.

<sup>21</sup>Assuming risk averse households would not change the results qualitatively, but it would increase the computational burden of the solution method as firms would have to keep track of aggregate consumption in order to determine their stochastic discount factor.

budget constraint

$$c_t = \mathcal{W}_t + bu_t + \mathcal{P}_t, \quad (1)$$

where  $c_t$  is aggregate consumption,  $\mathcal{W}_t$  is aggregate wage income,  $b$  captures home production and the value of leisure,  $u_t$  is the mass of unemployed workers and  $\mathcal{P}_t$  are aggregate firm profits.

### 4.3 Individual firm problem

Let us now consider the decision problem of a firm that chose to stay in the market in period  $t$ .

#### 4.3.1 Profit maximization

Active firms solve the following maximization problem

$$V^F(z_t, p_{i,t}, n_{i,t}) = \max_{n_{i,t+1}, v_{i,t}} [z_t p_{i,t} n_{i,t} - w_{i,t} n_{i,t} - \xi - \frac{\kappa_0 v_{i,t}^{\kappa_1}}{\kappa_1} + \beta(1 - \delta) \max\{0; E_t V^F(z_{t+1}, p_{i,t+1}, n_{i,t+1})\}] \quad (2)$$

s.t.

$$n_{i,t+1} = (1 - \rho_x)(n_{i,t} + q_{i,t} v_{i,t}), \quad (3)$$

where  $V^F(z_t, p_{i,t}, n_{i,t})$  is the period  $t$  value of an (active) firm with idiosyncratic productivity  $p_{i,t}$  and employment level  $n_{i,t}$ . Aggregate productivity is given by  $z_t$ ,  $n_{i,t}$  is firm level employment and  $w_{i,t}$  is the wage rate.  $\xi$  is an operational cost that needs to be paid at the beginning of the period and  $\kappa_0 v_{i,t}^{\kappa_1} / \kappa_1$  are the vacancy posting costs with  $v_{i,t}$  being the number of posted vacancies. Finally,  $\beta$  is the discount factor,  $\delta$  is an exogenous exit-inducing shock and  $E_t$  is the expectations operator taken over both aggregate and idiosyncratic productivity.

Firm value is composed of current profits (the difference between firm output and costs consisting of wages, vacancy posting costs and the operational cost) and the continuation value of remaining in operation. Each existing firm chooses whether to shut down or not at the end of period  $t$  (prior to observing period  $t+1$  shocks). Therefore, the continuation value cannot fall below zero as the firm chooses to shut down once expected firm value turns negative.

(3) is the law of motion for individual firm employment, where  $q_{i,t}v_{i,t}$  is the number of newly hired workers ( $q_{i,t}$  is the probability with which a posted vacancy gets filled). At the end of each period a fraction  $\rho_x$  of employed and newly hired workers get dismissed for exogenous reasons and they enter the unemployment pool.<sup>22</sup>

### 4.3.2 Firm output and vacancy posting costs

The production technology assumed in (2) uses labor as its only input and features constant returns to scale, as is done in many labor market model abstracting from capital.<sup>23</sup> Assuming a decreasing returns to labor technology would create an immediate link between firm growth, firm size and age. Young firms would tend to grow faster *automatically* only because they are born small, hence further away from their optimal size, implying higher marginal products of labor. This would contradict the findings of Haltiwanger, Jarmin, and Miranda (2010) that within a given age group smaller firms do not systematically grow faster than larger businesses. In the setting adopted in this paper, a small and a large firm with the same level of productivity will post the *same* number of vacancies.

A constant returns to labor technology, however, means that the concept of an optimal firm size vanishes. Nevertheless, the presence of exogenous worker dismissals and exogenous firm shut downs results in a well-defined firm size distribution.

Vacancy posting costs are assumed to be convex, as is commonly done in many models.<sup>24</sup> In the current context such an assumption implies that large, rapid, changes in employment are increasingly costly. In other words, firms that are productive enough to expand do so in a gradual manner.

### 4.3.3 Wages

Different models present different theories motivating wage setting. Some models base their wages on Nash bargaining, some on social norms, models with multi-worker firms often use the Stole-Zwiebel framework, while Kaas and Kircher (2011) introduce a new bargaining framework. In addition to the multiple theories that can stand behind wage

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<sup>22</sup>The assumption that new matches can get separated prior to production is made for convenience. It is straightforward to assume that new matches cannot separate prior to production (unless their employer shuts down), but the expressions get messier.

<sup>23</sup>See for example Moscarini and Postel-Vinay (2010b) or the classic Mortensen and Pissarides (1994) paper and many others that build on it.

<sup>24</sup>See for example Acemoglu and Hawkins (2010), Bloom (2009) Kaas and Kircher (2011), Merz and Yashiv (2007).

setting, it is also an open question how to calibrate the bargaining parameters, which often lack clear empirical counterparts.<sup>25</sup>

In this paper, I choose a reduced form approach and propose a simple wage setting rule. The wage rule parameters can then be calibrated such that the model delivers empirically plausible aggregate wage dynamics. The wage bill of a firm with productivity  $p_{i,t}$  is defined as

$$w_{i,t} = \tilde{\omega} z_t p_{i,t} + \omega p_{i,t} (1 - z_t). \quad (4)$$

In the steady state, wages are a fraction  $\tilde{\omega}$  of firm output. The parameter  $\omega$  controls the stickiness of wages with respect to aggregate productivity. The higher the  $\omega$ , the lower the positive response of wages to aggregate productivity. In this way, workers are rewarded relatively more in response to an increase in firm-specific efficiency compared to an increase in aggregate productivity common to all firms. The reason why the second term is not just a constant, as is the case other wage rule specifications, is that in the framework with idiosyncratic productivity shocks a fixed wage term would disadvantage relatively less productive firms.<sup>26</sup> For these businesses it would be relatively costlier to pay wages, while the wage rule presented here puts all firms on the same footing.

#### 4.3.4 Exogenous shocks

Aggregate productivity and firm-specific productivity of existing firms have the following processes

$$\log(z_t) = \rho_z \log(z_{t-1}) + \epsilon_t, \quad \epsilon_t \sim G_\epsilon, \quad (5)$$

$$\log(p_{i,t}) = \rho_p \log(p_{i,t-1}) + \eta_{i,t}, \quad \eta_t \sim G_\eta, \quad (6)$$

where  $\rho_z$  with  $\epsilon_t$  and  $\rho_p$  with  $\eta_t$  are autocorrelation coefficients and innovations of aggregate and idiosyncratic productivity, respectively.  $G_\epsilon$  and  $G_\eta$  are assumed normal with zero mean and standard deviations of  $\sigma_\epsilon$  and  $\sigma_\eta$ , respectively.

#### 4.4 Individual firm behavior

Let us now turn to the optimal decisions of individual firms.

<sup>25</sup>For instance Hagedorn and Manovskii (2008) show that an alternative calibration of the Nash bargaining framework can help resolve the “volatility puzzle” of standard matching models.

<sup>26</sup>For an example of a wage rule with a fixed wage term see den Haan and Lozej (2010)

#### 4.4.1 Vacancy posting

A firm posts vacancies until the costs of doing so are equal to the expected benefits. Define the value of a job at a firm with idiosyncratic productivity  $p_{i,t}$  as

$$J(z_t, p_{i,t}) = \frac{\partial V^F(z_t, p_{i,t}, n_{i,t})}{\partial n_{i,t}} = z_t p_{i,t} - w_{i,t} + \beta(1 - \delta)(1 - \rho_x) \max\{0; E_t J(z_{t+1}, p_{i,t+1})\}. \quad (7)$$

Given the functional form of vacancy posting costs the free-entry condition, which assumes that the value of posting a vacancy is pushed down to zero, reads

$$\kappa_0 v_{i,t}^{\kappa_1 - 1} = q_{i,t} \beta(1 - \delta)(1 - \rho_x) \max\{0; E_t J(z_{t+1}, p_{i,t+1})\}. \quad (8)$$

As is apparent from (8), firm size does not enter the vacancy posting condition and thus a small and a large firm with the same productivity level will post the same number of vacancies.<sup>27</sup> This property, together with the exogenous worker separation rate implies that small firms have relatively more resources left for expansion.<sup>28</sup>

#### 4.4.2 Firm entry

At the beginning of each period, *prior* to observing any period  $t$  shocks, potential firms can enter the economy. The cost of entering is given by  $\psi$  and it represents all the administrative, financial and utility costs related to starting up a new business. Paying this cost gives potential firms the chance to startup a business. Potential firms weigh this cost with the expected benefits of entry.

Upon entry new firms start with an employment level  $n_0$  and they obtain an initial idiosyncratic productivity draw  $p_{i,0} \sim G_{p,0}$ . Thereafter, the idiosyncratic productivity evolves (persistently) over time according to the law of motion in (6).  $G_{p,0}$  is assumed to

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<sup>27</sup>An alternative assumption is that vacancy posting costs are convex in the hiring rate ( $\kappa_0(v_t/n_t)^{\kappa_1}$ ) as for instance in Merz and Yashiv (2007). Such an assumption would imply that smaller firms post less vacancies than larger ones.

<sup>28</sup>For example, consider two firms, A and B, which have the same level of productivity, but firm A has 1,000 employees while firm B has 100 employees. Furthermore, assume that the firm-specific productivity implies that both firms will post 20 vacancies, assume that the exogenous worker separation rate is 10% and for simplicity assume that the probability of filling a vacancy ( $q_t$ ) is equal to 1. This means that while firm B expands by 10 employees (10 employees get exogenously separated, but firm B hires 20 new workers), firm A actually shrinks by 80 jobs (firm A also hires 20 new workers, but at the same time 100 employees exogenously separate).

be normal with a zero mean and standard deviation  $\sigma_{p,0}$  which is allowed to be different from  $\sigma_\eta$ . The initial productivity distribution is supposed to represent a wide range of “business ideas”. The distribution  $G_\eta$  then specifies how these business ideas evolve over time. It is therefore natural to think of the initial productivity distribution as being much wider than that of the *innovations* to firm-specific productivity. As becomes clear in the calibration section, this is actually the case.

The condition for firm entry can then be written as

$$\psi \leq \lambda_t E_{t-1} V^F(z_t, p_{i,0}, n_0), \quad (9)$$

where  $E_{t-1} V^F(z_t, p_0, n_0)$  is the expected value of a new firm (taken over both the aggregate and the idiosyncratic productivity level).  $\lambda_t$  is the probability of actually starting up a firm once the entry costs are paid. Entry occurs until (9) holds with equality.

The entry probability is assumed to depend on the mass of firms entering the economy ( $N_t^{New}$ ), reflecting two main effects. First, the possibility that not all potential firms that begin the process of starting up a new business actually finish or succeed.<sup>29</sup> Firms may not start up because of bad luck, discouragement, inability to obtain the appropriate documents or funds, etc. The reason why it depends on the number of entering firms is supposed to capture the notion of competition for funds needed to start a business, queues at the offices dealing with business start-ups, etc. Second,  $\lambda_t$  is also meant to capture the matching probability related to hiring an initial number of  $n_0$  workers. The higher the number of new firms, the lower the probability each firm has of hiring the desired number of workers. The entry probability is assumed to be given by  $\lambda_t = \alpha_0 (N_t^{New})^{\alpha_1}$ .

#### 4.4.3 Firm exit

At the beginning of each period, prior to observing period  $t$  shocks, incumbent firms decide whether to continue operating or to shut down.<sup>30</sup> The firm bases its decision on expected firm value.<sup>31</sup> If the firm chooses to exit, all its workers fall into unemployment. If, on the

<sup>29</sup>This does not relate to the fact that some start-ups are not productive enough. That is captured by endogenous exit of new firms.

<sup>30</sup>This assumption makes both the exit and the vacancy posting decisions depend on the expected value of the firm. In this way firms that decide to exit at the end of a period also did not post vacancies in that period.

<sup>31</sup>Note that firm profits can be negative if the value of staying in operation is large enough. Since the household is the owner of all firms, such losses then show up in aggregate firm profits in the household budget constraint.

other hand, the firm chooses to continue operating, it faces the optimization problem in (2).

Formally, one can define a cutoff point for firm-specific productivity,  $\tilde{p}(z_t, n_{i,t})$ , below which firms choose to exit. As aggregate productivity decreases, relatively more productive firms become so unprofitable that it no longer makes sense to stay in the market and they shut down. Similarly, a smaller firm will have a harder time generating enough revenue to be able to pay the operational cost. Hence, the cutoff point is inversely related to both the aggregate productivity shock and firm size. The cutoff point summarizing the firm's exit decision is implicitly defined by the following equation

$$E_{t-1}[V^F(z_t, \rho \tilde{p}(z_t, n_{i,t}) + \eta_{i,t}, n_{i,t})] = 0. \quad (10)$$

#### 4.5 The labor market and other aggregate variables

Let  $u_t$  be the mass of workers that are unemployed and let  $v_t$  be the mass of vacancies posted by all active firms in period  $t$ . Unemployed workers and vacancies match randomly on the labor market according to an aggregate matching function

$$M_t = \gamma u_t^\mu v_t^{1-\mu}, \quad (11)$$

where  $\gamma$  is match efficiency and  $\mu$  is the elasticity of matches with respect to unemployment. The choice of a Cobb-Douglas matching function with constant returns to scale follows common practice in the literature. The probability of a given unemployed worker finding a job is given by  $f_t = M_t/u_t$ . Similarly, the *average* probability of a firm filling its vacancy is given by  $q_t = M_t/v_t$ .

The total number of matches,  $M_t$ , is then “distributed” proportionally to individual firms based on their relative share in aggregate vacancies. This implies that all firms face the same probability of filling a vacancy. Define the number of matches of an individual firm as  $m_{i,t} = v_{i,t}q_{i,t}$ . One can then write

$$q_{i,t} = \frac{m_{i,t}}{v_{i,t}} = \frac{M_t \frac{v_{i,t}}{v_t}}{v_{i,t}} = \frac{M_t}{v_t} = q_t. \quad (12)$$

Let  $n_t$  be the mass of workers that are employed (producing) in period  $t$ . Then, the

law of motion for aggregate unemployment can be written as

$$u_t = 1 - n_t = u_{t-1} + \int_{i \in X_t^{exit}} \tilde{n}_{i,t-1} di + \int_{j \in X_t^{stay}} \rho_x \tilde{n}_{j,t-1} dj - M_{t-1} - N_t^{new} n_0, \quad (13)$$

where  $X_t^{exit}$  is the set of firms that were active in period  $t - 1$  but shut down in period  $t$ ,  $X_t^{stay}$  is the set of firms that are active in both period  $t - 1$  and  $t$ .  $\tilde{n}_{i,t}$  is the number of employment relationships in firm  $i$  at the end of the period and thus includes not only workers employed in period  $t$ , but also newly hired workers,  $\tilde{n}_{i,t} = n_{i,t} + m_{i,t}$  ( $n_{j,t}$  is defined analogously). Hence, the change in unemployment is given by the difference between the number of workers who exogenously separated or who were employed in firms that shut down and the number of unemployed workers who found jobs in existing or new firms.

Finally, aggregate vacancies and aggregate output are sums of the respective individual levels in active firms,  $v_t = \int_{j \in X_t^{stay}} v_{j,t} dj$  and  $y_t = \int_{j \in X_t^{stay}} n_{j,t} z_t p_{j,t} dj$ .

## 4.6 Equilibrium

The individual state variables of each firm are its productivity and employment,  $s_{i,t} = (p_{i,t}, n_{i,t})$ . The aggregate state is given by aggregate productivity and the joint cross-sectional distribution of firm-specific productivity and employment levels,  $\mathcal{F}_t(p_i, n_i)$ . The reason why the cross-sectional firm distribution is a state variable is because firms need to be able to predict the aggregate probability of filling a vacancy in order to be able to solve their maximization problem. This depends on the number of aggregate vacancies and unemployment. These variables are in turn determined by individual vacancy, employment, entry and exit decisions of all the firms in the economy. Thus, each firm needs to know the entire distribution of firm-specific productivity and employment levels in order to be able to predict the value of the probability of filling a vacancy.

Let  $S_t = (z_t, \mathcal{F}_t(p_i, n_i))$  denote the aggregate state. A competitive equilibrium is defined by

- (i) individual firm policy functions for employment,  $n(s_{i,t}, S_t)$ , vacancies,  $v(s_{i,t}, S_t)$ , and the exit decision,  $\tilde{p}(s_{i,t}, S_t)$ , that solve the individual firm problem in (2) and are consistent with the exit condition in (10),
- (ii) a mass of new entrants  $N^{New}(s_{i,t}, S_t)$  that satisfies the entry condition in (9),
- (iii) a joint distribution of firm-specific productivity and employment levels,  $\mathcal{F}_t(p_i, n_i)$ ,

that is determined by the (aggregate and idiosyncratic) exogenous productivity shocks and the employment, vacancy posting, exit and entry decision rules,

- (iv) individual firm wages and output are determined by  $n(s_{i,t}, S_t)$  and the exogenous productivity shocks,
- (v) and aggregate employment, vacancies, wages and output are given by the respective sums of individual firm variables over all active firms. Aggregate profits are the difference between total firm output and total costs consisting of wage, vacancy posting and operational costs, and aggregate consumption satisfies the budget constraint in (1).

## 5 Solution method

Maximization of the value function in (2) is not trivial, because among other things, one of the state variables is the cross-sectional firm *distribution*. This is a high-dimensional object and, more importantly, in the presence of aggregate productivity shocks, it varies over time. To deal with this issue, I follow Krusell and Smith (1998) and assume that firms track only a few moments of the idiosyncratic productivity distribution. In the next paragraphs I describe an iterative scheme that solves for the equilibrium. I refer to this scheme as the Krusell-Smith (KS) algorithm.

Next, I show that given the characteristics of the KS algorithm one can easily run into the curse of dimensionality. At the same time, a perturbation solution of the individual firm problem is not possible because of a non-differentiability in the continuation value. I thus propose to solve a *modified* maximization problem that allows for a perturbation solution for which higher dimensions of the problem pose no extra computational burden.

### 5.1 Krusell-Smith algorithm

The general idea of the KS algorithm is that instead of tracking the entire joint distribution of firm-specific productivity and employment levels, firms follow only a few of its moments. The resulting equilibrium is thus an approximate one, since some relevant information is left out. However, the choice of this set of moments is such that the resulting forecast errors from omitting other information are very small.

As is clear from the maximization problem, firms are ultimately interested in the aggregate vacancy filling probability,  $q_t$ . Thus, I let firms track the evolution of  $q_t$  *directly*,

rather than letting them follow moments of  $\mathcal{F}_t(p_i, n_i)$  and then relating these to  $q_t$ . The perceived law of motion for the aggregate vacancy filling probability is  $q_t = H(q_{t-1}, z_t, \mathcal{Q}_t)$ , where  $\mathcal{Q}_t$  is a collection of past aggregate productivity shocks and/or further lags of  $q$ . The knowledge of  $H$  allows firms to solve for their optimal behavior. The resulting decision rules can be used to simulate the economy, which delivers time series of simulated values of the vacancy filling probability. These can then be compared to those predicted by the perceived law of motion  $H$ . The resulting approximate equilibrium must be such that the goodness of fit is high. In other words the law of motion,  $H$ , must track the evolution of the simulated vacancy filling probability accurately.<sup>32</sup>

## 5.2 A modified maximization problem

From the exposition in the previous subsection it is clear that the dimensionality of the maximization problem can be quite high, if  $\mathcal{Q}_t$  includes many variables.<sup>33</sup> Solving the individual firm problem with, for instance, value function iteration becomes exceedingly computationally expensive.

Instead, one would like to solve the individual firm problem with perturbation techniques, in which case the high dimensionality of the maximization problem poses no extra computational costs. However, obtaining policy rules using perturbation is not possible, because the option to exit introduces a non-differentiability. To overcome this problem, I propose to solve a *modified* maximization problem, which is smooth and can therefore be solved using perturbation. Specifically, I replace the firm's objective function with the following Bellman equation

$$\begin{aligned} \tilde{V}^F(z_t, p_{i,t}, n_{i,t}) = \max_{n_{i,t+1}, v_{i,t}} [y_{i,t}(n) - w_{i,t}(n) - \xi - \kappa(v_{i,t}) + \\ \beta(1 - \delta)E_t \tilde{V}^F(z_{t+1}, p_{i,t+1}, n_{i,t+1})]. \end{aligned} \quad (14)$$

The above equation basically ignores the non-differentiability in the continuation value. Hence, the firm behaves as if it does not account for the option value of shutting down.<sup>34</sup>

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<sup>32</sup>Further details, such as a detailed description of the iterative procedure, the exact functional form of  $H$ , the composition of  $\mathcal{Q}_t$ , stopping criteria as well as accuracy tests are in Appendix A

<sup>33</sup>As is described in detail in Appendix A,  $\mathcal{Q}_t$  includes 5 variables and one therefore runs into the curse of dimensionality.

<sup>34</sup>Any losses incurred by the firm are transferred to the household as the owner of all firms. Furthermore, under risk neutrality of both firms and households and in the absence of a savings technology there are no precautionary motives that could alter agents' behavior.

Figure 6 illustrates the problem at hand. The true continuation value exhibits a non-differentiability where the zero lower bound kicks in. The perturbation solution to the modified maximization problem, however, just extrapolates into the negative part of the state-space.<sup>35</sup> Remember that the area of the state-space to the left of the “kink” is essentially irrelevant for continuing firms’ vacancy posting decisions. The reason is that if they find themselves in that part of the state-space, they choose to exit the market. What will be distorted, however, is the average value of the policy function. This value is lower in the case of the perturbation solution, since in some parts of the state-space the perturbation policy rule is negative, while the true policy rule is zero. However, as long as the dynamics of the respective decisions are not affected, the level difference poses no issues since the calibration targets realistic average values of firm closures and aggregate vacancies.

I will now look at a simple case which does not run into the curse of dimensionality and which I can solve accurately with value function iteration. In particular, I assume that the *true* law of motion for  $q_t$  depends only on  $z_t$ .<sup>36</sup> I then compare this accurate solution of the true problem (including the non-differentiability) to that obtained using perturbation techniques to solve the *modified* maximization. Figures 7 and 8 show de-meaned simulated exit and vacancy posting decisions based on the accurate solution of the true maximization problem (“true”) and those based on the perturbation solution to the modified maximization problem (“modified”). The two solutions yield very similar dynamics suggesting that using perturbation techniques on the modified maximization problem is a valid procedure.<sup>37</sup>

## 6 Calibration

To facilitate the calibration procedure, I divide the model parameters into two groups. First, parameters that are relatively standard or can be determined according to other studies in the literature, and second, parameters that are calibrated such that the model fits certain statistics in the data. Given that the aim of the model is to explain job creation and destruction patterns of firms of different age, targets related to this are not used in

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<sup>35</sup>During the simulation, however, I “bring back the kink” by making firms with negative continuation values exit the economy.

<sup>36</sup>Clearly this is a partial equilibrium setup, because the actual law of motion depends on a larger set of variables as is documented in Appendix A. However, for the purpose of checking accuracy of the solution to the individual firm problem, such a setup is sufficient.

<sup>37</sup>Further details on the modified maximization problem are presented in Appendix B.

the calibration. The only cross-sectional targets are the average firm size (of all and new firms), the average dispersion of employment growth (of all firms) and the average persistence of new jobs (in all firms). Table 5 summarizes all the parameter values.

## 6.1 Parameters taken from the literature

The first group of parameters contains the discount factor,  $\beta$ , the standard deviation of aggregate productivity innovations,  $\sigma_z$ , the autocorrelation coefficient of aggregate productivity,  $\rho_z$ , match elasticity,  $\mu$ , the exogenous separation rate,  $\rho_x$ , the exogenous firm exit probability,  $\delta$ , the value of home production and leisure,  $b$ , and the power in the vacancy posting cost function,  $\kappa_1$ .

$\beta$  is set to 0.99 as the model period is assumed to be one quarter,  $\sigma_z$  is set to 0.007 and  $\rho_z$  is fixed at 0.95 as is standard in the literature.  $\mu$  is set to 0.72 following the recommendation of Shimer (2007). Helfand, Sadeghi, and Talan (2007) report that in the BDS database through years 1990 to 2005 17.2% of gross job loss was due to firm closures. This is similar to the value of roughly 1/6 reported in Davis, Haltiwanger, and Schuh (1996), for manufacturing firms.  $\rho_x$ , is thus set to  $(1 - 0.172)\rho_{total}$ , where  $\rho_{total}$  is the total separation rate. The calibration related to the total separation rate is explained in the next subsection. Helfand, Sadeghi, and Talan (2007) also document that firm closures account for 1.7% of gross job loss at firms with more than 1,000 employees. The model in this paper predicts that the endogenous exit probability of large firms is virtually zero (large firms are also more productive and thus they first go through periods of contraction before they endogenously exit). Therefore,  $\delta$  is set such that  $\delta = 0.017(\rho_x + \delta)$ . Without loss of generality  $b$  is normalized to zero. Because of the structure of the adopted wage rule in (4) the outside option of workers does not enter wages. In the current setup it only affects the scale of aggregate output. Finally, the vacancy cost function is assumed to be quadratic in vacancies, i.e.  $\kappa_1 = 2$ , following Acemoglu and Hawkins (2010).<sup>38</sup> Given this value (and the value of  $\kappa_0$ , the calibration of which is described in the next section) costs of a newly hired worker in an average firm are 8.7% of output per worker.<sup>39</sup>

<sup>38</sup>The choice of quadratic vacancy posting costs is somewhat arbitrary and therefore I also investigate the case when  $\kappa_1 = 5$ . The main results of the paper still hold under this alternative calibration.

<sup>39</sup>The total vacancy posting costs amount to roughly 17% of output per worker in an average firm. Each period there are  $qv$  workers hired. This means that it takes  $17/(vq) \approx 8.7\%$  of output per worker to hire one new employee in an average firm.

## 6.2 Parameters chosen to match statistics in the data

The second group of parameters that are calibrated to match statistics in the data consists of match efficiency,  $\gamma$ , the scale parameter in the vacancy cost function,  $\kappa_0$ , the operational cost,  $\xi$ , the entry cost,  $\Psi$ , the scale parameter in the entry probability function,  $\alpha_0$ , the power parameter in the entry probability function,  $\alpha_1$ , the two wage rule parameters  $\tilde{\omega}$  and  $\omega$ , the standard deviation of individual firm productivity innovations,  $\sigma_p$ , the auto-correlation coefficient of individual firm productivity,  $\rho_p$ , the standard deviation of the initial firm productivity draw,  $\sigma_{p,0}$ , and initial employment size,  $n_0$ . These nine parameters are set such that the model is able to match nine statistics in the data. To ease the exposition, I group the statistics into three categories: first-order moments of aggregate variables, second-order moments of aggregate variables, and firm level statistics.

**Targeting aggregate first-order moments.** This category of parameters consists of  $\gamma$ ,  $\kappa_0$ ,  $\xi$ ,  $\Psi$  and  $\alpha_0$ . These parameters are set such that the model delivers a steady state unemployment rate of 12%, as for instance in den Haan, Ramey, and Watson (2000) and Krause and Lubik (2007). Following den Haan, Ramey, and Watson (2000) and van Ours and Ridder (1992) the model targets a vacancy filling probability of 71%. The model further targets a total separation rate of 10% which is typically done in the literature. This value is based on evidence by Hall (1995) and Davis, Haltiwanger, and Schuh (1996) and is used for example by den Haan, Ramey, and Watson (2000) and Krause and Lubik (2007).<sup>40</sup> Without loss of generality,  $\Psi$  is normalized to 1 and  $\alpha_0$  is calibrated such that in the steady state entry equals exit.<sup>41</sup>

**Targeting aggregate second-order moments.** This group of parameters includes  $\alpha_1$ ,  $\tilde{\omega}$  and  $\omega$ . These parameters are set such that they fit three second-order moments of aggregate variables relative to the standard deviation of (log) labor productivity. The three statistics are the relative volatility of the share of new firm employment in total employment equal to 0.24, the relative employment rate volatility of 0.44, and the relative volatility of (log) wages equal to 0.76.

**Targeting cross-sectional statistics.** Finally,  $\sigma_p$ ,  $\rho_p$ ,  $\sigma_{p,0}$  and  $n_0$  are set such that the model matches four cross-sectional statistics. First, the average dispersion of firm employment growth rates of roughly 0.56 as reported in Davis, Faberman, Haltiwanger, Jarmin, and Miranda (2008). This dispersion measure is the average cross-sectional standard deviation of firm employment growth rates, including entry and exit over the period

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<sup>40</sup>This implies a value of 8.28% for the exogenous separation rate  $\rho_x$ .

<sup>41</sup>The entry condition in (9) shows that  $\alpha_0$  and  $\Psi$  cannot be identified separately.

between 1992 and 2007. Second, the average persistence of new jobs is targeted to be 0.68 as documented in Davis, Haltiwanger, and Schuh (1996). This value gives the fraction of newly created jobs that survive into the next year.<sup>42</sup> Finally, the model targets the average firm size and the average size of new firms to be 21.3 and 6.1, respectively, as found in the BDS database.

## 7 Model results

The main goal of this paper is to build a theoretical framework to help us understand the role of firm age in determining firm growth and its importance for aggregate labor market dynamics. In this section, I document that the model in this paper correctly predicts that both firm age and firm size are negatively related to the growth rate of firms as in the data. However, once age and size are taken into account together, the significance level of the negative coefficient on firm size drops as documented by Haltiwanger, Jarmin, and Miranda (2010).

Furthermore, this section shows that the model is consistent with the other empirical findings presented in Section 3 and that it predicts realistic aggregate labor market dynamics, both in terms of co-movement and volatility.

The reported results are based on values from 1,000 simulations. Each simulation has 1,108 quarters, where the first 1,000 are dropped to obtain 108 quarters as a counterpart to the sample used in the empirical part.

### 7.1 Size, age and firm growth

Table 6 reports model-based regression results of firm level employment growth rates on age, size and age and size together. The upper panel, comparable to Table 4 in Haltiwanger, Jarmin, and Miranda (2010), shows results for a specification which uses dummy variables for age and size groups.<sup>43</sup> The coefficients can then be interpreted as averages in the respective age/size category.

Employment growth rates monotonically decline with firm age (when firm size is not considered) as documented in the data. Furthermore, a negative relationship can also be seen between firm size and business growth (when firm age is not considered), although to a lesser extent, since the largest firms are basically stagnant.

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<sup>42</sup>It does not refer to individual jobs, but rather to an increase in employment that persists for one year.

<sup>43</sup>In the specification where age and size are used together, the category of the largest firms is excluded.

The most interesting case, however, is when both age and size are used together as regressors. In this case, age still retains its negative relationship with respect to firm employment growth rates. However, the inverse relationship between firm size and its growth is considerably weakened. The clearest example is the group of very small firms for which the average growth rate (compared to the largest group) is significantly negative.

Although informative, the age and size categories are somewhat arbitrary. Therefore, the lower panel reports results from a regression where age and size are not grouped into categories. In this case I regress firm employment growth rates on their age and size directly, and not on the respective dummy variables. The above-mentioned patterns are confirmed by these regression results. Both age and size alone display a negative, statistically significant, coefficient. However, when considering age and size together, the significance level of the coefficient on size drops (the t-statistic falls from  $-10.8$  when only size is considered to  $-2.2$  when both age and size are taken into account). Moreover, the magnitude of the coefficient on size drops to one quarter of its value when age is not considered, while the coefficient on age hardly changes across the different specifications. Although the coefficient on firm size is still significantly negative, the model comes close to capturing the recent finding that it is mainly age that matters for firm growth, not size.

The negative relationship between firm age and firm growth is driven by the effect of firm entry and the evolution of firm-specific productivity. Some business start-ups initially have high productivity levels, but as they grow old, they revert back to the (common) mean of firm-specific productivity. Moreover, given a productivity level a small and a large firm will hire the same number of workers. However, smaller firms exogenously lose less workers (in absolute terms) leaving them with relatively more resources for expansion compared to larger businesses. Therefore, given that young firms are mainly small, the combination of these effects means that young businesses grow faster than older ones.

When controlling for firm age, the group of small firms no longer includes the effect of business start-ups. Moreover, young firms are characterized by a wider right tail of the firm-specific productivity distribution and this positive effect on firm growth is also conditioned away when firm age is taken into account. For these reasons the negative coefficient on firm size loses considerably on statistical significance when firm age is included into the regression. However, the effect of smaller firms having to use less resources for covering the gap after exogenously dismissed workers before expanding is still present.<sup>44</sup>

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<sup>44</sup>As discussed earlier, assuming vacancy posting costs are convex in the hiring rate ( $v_t/n_t$ ) would make smaller firms post less vacancies. In this case, the above-mentioned effect would be weakened and the

## 7.2 Other model predictions related to firm age

In the following paragraphs I document that the model is also consistent with the other empirical findings presented in Section 3.

### 7.2.1 Firm age/size distribution

Even though the calibration does not target firm age shares, the nature of the firm growth process is such that the model predicts that 38% of all firms are young. This is very close to the 41% found in the data.

Furthermore, in reality the majority of young firms are small, while the converse is less true. Figure 9 shows the firm age shares in a given group of small firms.<sup>45</sup> The model overpredicts the age shares for the very young at the expense of old firms, but the shares do *not* die out with age. Hence, it is not the case that small firms are mainly young. Figure 10 plots the size shares of new entrants, firms younger than 2 years and firms younger than 6 years. All three groups are predominantly small as in the data.<sup>46</sup>

The results are driven by the fact that young firms are born small and conditional on survival they gradually expand. However, the linear production technology does not restrict older (larger) firms from contracting if their firm-specific productivity falters.

### 7.2.2 Exit rates, job creation and destruction according to firm age

**Exit rates.** Figure 11 depicts the empirical and model-based exit rates as a function of firm age. The negative relationship between firm exit rates and age is present, but it is weaker, especially after the first year of a firm's life. The model exaggerates the exit rate of new firms while it underpredicts the probability of shutting down for all the other age groups. This is driven by the selection process of successful firms. The relatively less productive businesses shut down early in their lives leaving only the more efficient ones to grow old. In the model, this process is relatively strong and therefore exit rates fall sharply after the first periods of a firms' life.<sup>47</sup>

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model could get even closer to the findings in Haltiwanger, Jarmin, and Miranda (2010).

<sup>45</sup>In the data firms with 1 to 4 workers account for about 43%. The model size distribution is not entirely comparable with the empirical one due to the absence of very large firms. Thus, the figure compares the empirical age shares of firms with 1 to 4 employees, to the age shares of the smallest 43% of all firms in the model.

<sup>46</sup>Empirical counterparts are not shown because, as mentioned, the model size distribution is not entirely comparable to that in the data.

<sup>47</sup>An important driving factor of the strong selection in the group of new firms is the assumption that initial productivity is drawn after entry. If, instead, one would assume a certain degree of pre-selection

**Job creation and destruction rates.** Figures 12 and 13 show the empirical and model-based job creation and destruction rates according to firm age. The model qualitatively captures the declining job creation and destruction rates according to age, but it underpredicts their levels (except for new firms).

The declining nature of job creation and destruction rates with firm age is again driven by the selection process of successful firms. This, in combination with the effect of firm entry, results in young firms having higher (net) job creation rates than older businesses. The relatively low job destruction rates of older firms are driven by their low exit rates.

**Job creation and destruction shares.** In the data, 15% of all workers are employed at young firms. At the same time young firms account for 34.3% and 21.5% of all gross job creation and destruction, respectively. In the model, young firms account for 18.5% of employment, 54.5% of all gross job creation and 19.6% of all gross job destruction. Hence, as in the data, young firms play an important role in job creation since they create a disproportionately large amount of (net) jobs compared to their employment share. The model exaggerates the share of young firms in job creation, which is related to the underpredicted level of job creation rates of older firms. Thus, although the model underpredicts the extent of job reallocation (the sum of gross job creation and destruction), it performs relatively well in capturing the importance of young firms in this process.

### 7.2.3 Cyclical behavior of firm-level employment

The model is consistent with the procyclical behavior of employment growth rates of young firms (correlation coefficient with the unemployment rate of  $-0.35$ ). As aggregate productivity increases, the incentives to post vacancies rise and the risk of shutting down falls. Moreover, these incentives are not dwarfed by the costs of replacing exogenously separated workers as young firms are mainly small.

The correlation coefficient of unemployment and the employment growth rate of old firms, however, is virtually zero at  $-0.06$ . The reason for this is twofold. A large part of old firms are productive and therefore also large. For these firms the additional incentives for hiring brought up by an increase in aggregate productivity are dwarfed by their large costs of replacing exogenously dismissed workers. Those old firms that are relatively small and could benefit more from an increase in aggregate productivity, however, are also relatively unproductive and face a high risk of shutting down. Hence, a boom provide extra  

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prior to entry, exit rates of new firms would be dampened.

incentives to hire only for those old firms that are not too large but that are productive enough to expand.

Because young firms have higher job creation and destruction rates on average, young businesses are also more volatile than older ones. The standard deviation of the employment growth rate of young firms is 3.5 times larger than that of old firms. This is slightly higher than in the data. Moreover, the model is consistent with the negative (albeit a slightly weaker) correlation between the differential employment growth rate of young and old firms with the unemployment rate (correlation coefficient of  $-0.32$  compared to  $-0.62$  in the data).

#### 7.2.4 Importance of entry

Section 3 documented that firm entry is important for aggregate dynamics. One can also zoom in on new firms in the model.<sup>48</sup> Starting at the steady state, Figure 14 shows the impulse response function (IRF) of the unemployment rate to a negative one-standard-deviation shock to aggregate productivity. The IRF is then decomposed into the contribution of firm entry and the combined contribution of exit, job creation and destruction of continuing firms (“rest”).

Upon impact lower firm entry accounts for almost 40% of the unemployment increase. Moreover, firm entry remains below its steady state level long after the initial hit because of persistently lower expected benefits of starting up a business. Remember that new firms enter based on the expected value of individual firm productivity (not a realization) and they obtain a “business idea” only after entry. During periods of lower aggregate productivity, there is an increase in the chance of the initial draw not being high enough in order for the firm to survive in the market, reducing the incentives for entry.

The combined contribution of exit, and net job creation of continuing firms reverts within a few periods and starts to push unemployment down to its steady state. This is because the larger unemployment pool makes it easier for existing firms to hire workers. Note that potential firms also take this into account as firm value depends on the aggregate probability of finding a worker. However, this effect is not strong enough to overturn the

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<sup>48</sup>The typical way to simulate is to use a large (finite) number of firms. However, the group of new firms constitutes only a small fraction of the total number of firms and thus one needs to be concerned with sampling uncertainty even if the total number of firms is large. To this end I use a non-stochastic cross-section simulation method. This grid technique does *not* feature cross-sectional sampling uncertainty and is thus suitable for this purpose (details are provided in Appendix C)

negative impact on incentives for entry caused by the higher expected exit rates.<sup>49</sup>

### 7.3 Model predictions related to aggregate variables

In this section, I analyze the implications of the model for aggregate variables. First, I examine impulse response functions to an aggregate productivity shock. The IRFs of the presented heterogeneous firm (HF) model are compared to those obtained from a representative firm (RF) matching model using the same calibrated parameters. Second, I examine the business cycle properties of aggregate labor market variables predicted by the HF model and compare them to those observed in the data.

#### 7.3.1 Impulse response functions

Heterogeneity in firm-specific productivity levels together with the endogenous process of firm entry, growth and exit create two new propagation channels that are not present in a standard matching model. In the HF model the *number of firms* varies procyclically and *average firm productivity* fluctuates countercyclically. These two opposing effects result in new interesting dynamics as well as greater propagation.

Figure 15 shows the impulse responses of unemployment, vacancies, the probability of filling a vacancy and output to a positive one-standard-deviation shock to aggregate productivity generated by both the HF and RF model. The response of output in the HF model is further decomposed into the effect of employment, average firm productivity and aggregate productivity. The middle right panel shows impulse responses of the number of active firms and the cutoff value  $\tilde{p}(z_t, n_0)$ , representing the probability of shutting down (these do not have counterparts in the RF model).<sup>50</sup>

First, the HF model is characterized by greater propagation as the responses of vacancies and the probability  $q$  die out more gradually than in the RF case. The response of unemployment and the probability of filling a vacancy displays a larger drop in the HF model compared to the RF one. This is due to endogenous firm entry and exit. As aggregate productivity improves less firms exit and more firms enter the economy pushing down unemployment further. This directly translates into a stronger decline of the probability of finding a worker as the pool of available unemployed is relatively smaller than in the

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<sup>49</sup>Remember that the initial productivity distribution is wide and therefore changes in aggregate productivity affect a relatively large mass of new firms.

<sup>50</sup>Remember that the cutoff value depends on both aggregate productivity and employment. The IRFs of cutoff values for different firm sizes are qualitatively similar.

RF case. This stronger drop in the chances of hiring new workers provides relatively less incentives to post vacancies and thus their response is weaker compared to the RF model.

Second, the response of vacancies does *not* mimic that of the exogenous shock in the HF case. Instead, it displays a hump-shaped response as is the case in the data.<sup>51</sup> The reason for this is the hump-shaped response of the total number of active firms in the economy. As survival rates increase in reaction to a positive aggregate productivity shock the new entering firms thus meet with more and more existing firms which have survived from previous periods. Hence, the total number of firms gradually cumulates before returning to its steady state level.

Third, while the response of output in the RF model gradually declines with aggregate productivity, in the HF model it has intriguing dynamics. After roughly 4 years, output actually increases slightly before returning to its steady state. There are two forces at play. As aggregate productivity increases the risk of shutting down declines. This results in relatively less productive firms being able to stay in the market reducing average firm productivity. At the same time, more firms enter the economy, a large part of which are highly productive. However, the relatively less productive incumbents that can afford to stay because of the lower risk of shutting down are (on average) larger than business start-ups. Together with the fact that the cumulation of new productive firms takes time, this implies that initially the negative effect of higher survival rates of relatively less efficient firms dominates and average firm productivity falls. After exit rates have returned back to their steady states and young productive firms have grown in size, the positive effects of a larger fraction of (highly productive) young firms takes over and average firm productivity rises. This leads to an increase in output which eventually returns back to steady state together with the total number of firms.

### 7.3.2 Business cycle properties

Table 7 summarizes the standard deviations and correlations of selected labor market variables in the U.S. economy. Table 8 reports the same statistics generated by the HF model. For comparison, Table 9 reports the business cycle statistics generated by the standard matching model assuming a representative firm. This time, however, the RF model is recalibrated to fit the same statistics as the HF model that are common to both

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<sup>51</sup>The response of vacancies mimicking the dynamics of the exogenous shock is counterfactual to the data but stubbornly robust in standard matching models as pointed out by for instance Fujita and Ramey (2006).

models.

The HF model captures the observed volatility and autocorrelations of labor market variables well. The key reason behind the ability to capture the respective volatilities is that the surplus share of the production relationship is small for the firm. This is the result of the calibration of the wage setting rule. A small firm surplus share implies that aggregate productivity shocks have a larger effect on the value of jobs for a firm and thus on the vacancy posting incentives as pointed out in Hornstein, Krusell, and Violante (2005). As explained in the previous section, firm heterogeneity together with the endogenous process of firm entry and exit generate greater persistence allowing the model to capture well the observed autocorrelation coefficients.

The only exception where the model noticeably underpredicts volatility is labor market tightness (and hence the job finding probability). This is related to the weaker (negative) correlation between vacancies and unemployment (the Beveridge curve). The reason is that in a boom less firms are forced to shut down and more firms enter, which reduces the pool of unemployment available for hiring. This in turn lowers the probability a given vacancy is filled and thus the incentives to post vacancies are diminished. Similar logic applies to recessions. The model captures the rest of the correlation structure well. Hence, overall the model does a good job in generating realistic aggregate labor market dynamics, both in terms of co-movement and volatility.

The RF model strongly overstates the volatility of vacancies (and thus also labor market tightness and the job finding rate) for reasons explained in the previous section. Moreover, it performs relatively worse in capturing the autocorrelation coefficients compared to the HF model.

## 8 Government policy supporting young firms

The presented framework stresses the importance of firm heterogeneity for aggregate outcomes, especially the dimension of firm age. It thus provides an ideal laboratory in which to analyze questions relating to the role of young firms. Such questions are gaining on importance as unemployment rates remain stubbornly high in many countries affected by the most recent crisis and governments struggle to boost employment growth. For instance, the recent “Startup America” initiative of the White House is partly aimed at supporting young firms and consists of four main points: (i) easing access to capital, (ii) reducing regulatory barriers, (iii) providing mentoring and advice and (iv) tax relief and

incentives.<sup>52</sup> In this section, I first argue that there is a role for government intervention in the presented model. I then show the effects of a government policy aimed at supporting young firms, similar to the Startup America program.

Under the presented model calibration, the wage setting rule stipulates output shares for workers and firms that are far away from the efficient ones dictated by the Hosios condition (in the steady state the firm gets 3% of output). In such a case, the economy is characterized by “underinvestment” resulting in too little firm entry and too much firm exit.

The above suggests that there is a role for government intervention in the form of supporting business operations. For instance, subsidizing the operational costs  $\xi$ , which does not show up in the wage setting rule, raises firm profits, increases the resources for hiring new workers and lowers the risk of a subsidized firm shutting down. The majority of firm selection takes place early on in a firms’ life (young firms exhibit high exit rates) justifying the consideration of a subsidy aimed at young firms.

Specifically, I assume that firms pay only a fraction  $1 - \tau$  of their operational cost, where  $\tau$  is the government.<sup>53</sup> I consider two different implementations of the government policy. First, only firm entry is subsidized leaving the behavior of existing firms unchanged. Second, only firms older than one quarter are subsidized with the subsidy dying out monotonically until the time the firm reaches 3 or 5 years of age. For comparability, in all cases the total per period amount spent on the subsidy is equal to 1% of steady state aggregate output in the case of no subsidy.

Table 10 summarizes the results, where the reported values are relative to the case with no government intervention. In the case when only firm entry is subsidized (first column) the government policy leads to higher output (the reported values for output are net of the costs of the subsidy) and thus increases welfare.<sup>54</sup> With more firms entering the economy unemployment decreases and average firm productivity rises as many of the new firms are highly productive. At the same time, however, the average exit rate increases, since a larger fraction of new firms comes also with relatively more firm closures.

Under the second scenario (second and third column) only existing firms are subsidized. For both considered durations of the subsidy unemployment declines, but output actually

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<sup>52</sup>For more information see <http://www.whitehouse.gov/issues/startup-america>.

<sup>53</sup>Further details and issues related to the technical implementation of the subsidy in the solution of the model are in Appendix D.

<sup>54</sup>One does not need to be concerned by fluctuations when examining the welfare in the presented model, since all agents are assumed to be risk neutral.

*falls*. The reason behind this lower output level is a fall in average firm productivity. The subsidy on existing firms lowers the exit rates of active firms enabling the relatively less productive ones to remain in business. Unemployment falls as less workers lose their jobs due to firm closures lowering the probability of filling a vacancy. This reduces incentives for entry and less businesses startup.<sup>55</sup> Lower firm entry comes with a relatively lower mass of highly productive firms. Thus, the combination of higher survival rates of relatively less productive businesses and lower firm entry results in a fall in average firm productivity.

Moreover, the process of firm closures releases workers from relatively unproductive firms and make them available to be hired by more productive businesses. This reallocation process is made less efficient when the government subsidizes existing firms. Such an adverse effect is not present in the case of subsidizing firm entry as there is no firm selection at that stage of a firms' life. Only upon entry do firms obtain their initial productivity draws.

The model thus predicts that the most important part of policies such as the Startup America program is getting rid of barriers to entry. Once new firms can easily startup, government support should be swiftly withdrawn as relatively inefficient incumbent firms crowd out potentially highly productive entrants and the worker reallocation process is disrupted.

## 9 Conclusion

Recent research has shown that it is not *small* firms that grow faster and are important for job creation in the aggregate economy, but rather *young* businesses. This questions the current way of thinking that has treated firm size as an important determinant of business growth. This paper turns its attention to firm age as the driver of firm growth and examines the role of young businesses in determining aggregate labor market dynamics.

It has been documented that young businesses have higher exit rates and grow faster than older ones, job creation and destruction rates fall with firm age and that young firms create relatively more (net) jobs. I extend these findings by documenting that young businesses are also more volatile than older ones and that firm entry is important for unemployment rate developments.

To further understand these relationships, I build a novel general equilibrium model

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<sup>55</sup>Note that new firms also take into account the increased survival rates of older firms. This effect dampens the decline in firm entry, but does not overturn it.

with labor market frictions and heterogeneous firms that is consistent with the above facts and produces realistic dynamics of aggregate labor market variables. Firm heterogeneity together with the endogenous process of business start-ups, firm growth and firm closures create a new propagation mechanism resulting in greater persistence and more complex dynamics than otherwise present in a standard matching model. The presented framework also enables us to study policy questions related to firm age. The results suggests that governments should mainly focus on easing barriers to entry. Supporting existing firms disrupts the selection process of successful firms leading to lower average firm productivity and in turn lower output.

Although this framework performs well in terms of capturing the firm level characteristics present in the data, and provides us with a laboratory to study new policy questions, some aspects of the model still deserve more attention. In particular it would be especially interesting to allow workers to search for new jobs while being employed or to let them quit. This would then alter the worker reallocation process, since in the presented model workers can change employers only following an exogenous dismissal, or when their current employer shuts down.

Furthermore, the model indicates that the number of business start-ups is a crucial aspect to overall job creation. Moreover, the policy exercise documented that new firms are key in not only creating jobs, but also increasing productivity. Further analysis of the entry decision supported by empirical evidence related to characteristics of new firms could sharpen the policy implications of the model.

Finally, there is a potentially important issue concerning young firms that has been left out completely in this paper. Namely, the importance of financial frictions for young businesses. As a new firm, it can be much harder to obtain external funding, for instance because entrants cannot prove themselves with a successful credit history. Incorporating financial frictions could strengthen the results of the presented model, as the firms that are responsible for a large part of job creation in the economy could also be those that are hit hardest by financial frictions. Large propagation effects could arise as the worsened conditions for obtaining external funds during recessions would further discourage firm entry. These and other extensions, however, are left for future research.

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## Appendix

### A Krusell-Smith algorithm - details and accuracy checks

In this section I provide details on the exact specification of the aggregate law of motion of the aggregate vacancy filling probability,  $H$ . Also, details of the simulation, stopping criteria and accuracy checks are shown.

#### A.1 Details on solution algorithm and simulation

The KS algorithm consists of the following iterative procedure

- 1. select  $\mathcal{Q}_t$
- 2. guess a functional form for  $H$  and values of its coefficients
- 3. given  $H$  solve the individual firm problem
- 4. given the decision rules from step 3, simulate the economy and obtain a simulated time series of the vacancy filling probability,  $q_{sim}$
- 5. use the simulated time-series from step 4 and estimate parameters ( $H_{new}$ ) of the aggregate law of motion for  $q$
- 6. compare  $H_{new}$  and  $H$ . If the parameters are not close to each other, update the guess of  $H$  using  $H_{new}$  and go back to step 3. If the parameters have converged, but the goodness of fit is low, increase the number of lags of  $z$  and/or the number of lags of  $q$  in  $\mathcal{Q}_t$  and go back to step 3. Alternatively, assume a different functional form for  $H$  and go back to step 3. If the parameters have converged and the goodness of fit is high, stop

As mentioned in the main text, the aggregate law of motion is summarized as  $q_t = H(q_{t-1}, z_t, \mathcal{Q}_t)$ , where  $\mathcal{Q}_t$  is a collection of (past) aggregate productivity shocks and/or moments of the idiosyncratic productivity distribution and further lags of the aggregate probability of filling a vacancy. The choice of  $\mathcal{Q}_t$  is a balance between a parsimonious specification and accuracy.

In the current setup  $H$  is assumed to be log linear and  $\mathcal{Q}_t = (z_{t-1}, z_{t-2}, q_{t-2}, q_{t-3})$ . The reason why lagged values of the aggregate productivity shock are useful for predicting

aggregate labor market tightness (and thus the aggregate probability of filling a vacancy) is that in the end it is aggregate productivity that drives fluctuations in the *distribution* of individual productivities. Alternatively, one could also use moments of the firm productivity distributions, such as the mass of firms at certain quantiles, the dispersion etc. However, these moments will necessarily be arbitrary choices and they will themselves become state variables that will likely depend on lagged values of aggregate productivity.

Given coefficient values in the aggregate law of motion,  $H$ , the individual firm problem can be solved and the model can be simulated. The simulation is done with 200,000 firms (increasing the number of firms does little to the results). The model is then simulated for 5,000 periods, where the first 1,000 are dropped. The remaining 4,000 periods are used to update the coefficients in  $H$ .

The stopping rule is based on the maximum absolute percentage difference between the coefficients in  $H$  used to solve the firm problem and those that come out of the regression using simulated time-series. The stopping criterion is  $10^{-6}$ . Finally, new coefficients are updated with a dampening factor of 0.25. This means that the new coefficients in  $H$  are a weighted average of regression coefficients and the coefficients from the previous iteration, where the weight on the regression coefficients is 0.25.

## A.2 Accuracy checks

Figure 16 shows an accuracy plot which compares the simulated time path of  $q$  (“actual”) and the simulate path based on the aggregate law of motion (“aggregate law of motion”). The time path based on the aggregate law of motion is given by

$$q_t^{alom} = H(q_{t-1}, z_t, z_{t-1}, z_{t-2}, q_{t-2}^{alom}, q_{t-3}^{alom}), \quad (15)$$

where the superscript *alom* indicates that the series is based on the aggregate law of motion given by  $H$ . Notice, that  $q_t^{alom}$  does *not* use the actual value of the vacancy filling probability as one of its inputs and thus the approximation errors are allowed to cumulate. The figure shows that the two time paths are very close to each other.

Looking at the average percentage difference between the two time series shows only a very small average difference of 0.04%. However, this could still hide large difference. Therefore, I also consider the maximum percentage differences, which in this case is 0.26%. This value occurs during a very sharp drop in aggregate productivity (a fall from 0.02 to  $-0.005$  which is a change corresponding to almost 3 times the standard deviation of the

innovation to  $z$ ). Excluding this extreme drop the maximum percentage difference falls to 0.14% a very low value.

Finally, the impulse responses of aggregate variables based on using the simulated value for  $q$  and that based on the aggregate law of motion are virtually identical to each other.<sup>56</sup> The above evidence suggests that the aggregate law of motion for  $q$  does a very good job at approximating the actual (simulated) behavior of  $q$ .

## B A modified maximization problem

As is documented in the previous section, the dimensionality of the individual firm maximization problem is quite high due to the KS algorithm. Apart from the “typical” state variables (individual firm productivity, firm employment and aggregate productivity) firms also take into account 5 lags of the aggregate productivity level as well as one lag of  $q$ . The latter help predict the value of the aggregate vacancy filling probability needed in the firms’ maximization problem.

In this section I compare two solutions to the individual firm problem presented in this paper. For convenience I replicate the equations of the maximization problem below. An individual firm maximizes its value, subject to the law of motion for firm employment

$$V^F(z_t, p_{i,t}, n_{i,t}) = \max_{n_{i,t+1}, v_{i,t}} [y_{i,t}(n_{i,t}) - w_{i,t}(n_{i,t}) - \xi - \kappa(v_{i,t}) + \beta(1 - \delta) \max\{0; E_t V^F(z_{t+1}, p_{i,t+1}, n_{i,t+1})\}] \quad (16)$$

s.t.

$$n_{i,t+1} = (1 - \rho_x)(n_{i,t} + q_{i,t}v_{i,t}). \quad (17)$$

In the main text the firms do not actually maximize (16), but rather a modified objective function of the following form

$$\tilde{V}^F(z_t, p_{i,t}, n_{i,t}) = \max_{n_{i,t+1}, v_{i,t}} [y_{i,t}(n) - w_{i,t}(n) - \xi - \kappa(v_{i,t}) + \beta(1 - \delta) E_t \tilde{V}^F(z_{t+1}, p_{i,t+1}, n_{i,t+1})]. \quad (18)$$

---

<sup>56</sup>Results upon request.

As mentioned in the main text, the policy function based on the perturbation solution to the modified maximization problem will have a lower mean. However, as long as the dynamics of the firm decision rules are not affected, the level difference poses no issues, since the calibration targets realistic average values of firm closures and aggregate vacancies.

The goal of this section is to document how close are the solutions to the modified and the true maximization problems in terms of dynamics. In order not to run into the curse of dimensionality, which is the original reason to use perturbation techniques, I consider a simplified version of the aggregate law of motion for  $q$ . Namely,  $H$  only consists of only the current aggregate productivity state. All other parameter values are as in the main text. Note that this is a valid procedure, since I am interested only in the accuracy of the individual firm problem and I am not solving for the general equilibrium.

A discrete approximation was made for the exogenous Markov processes for individual and aggregate productivity. While individual productivity was approximated using 100 grid points, only 20 grid points were used for aggregate productivity. A higher number of grid points for individual productivity ensures enough accuracy for the firm exit decision. Furthermore, a grid of 250 points was created for firm level employment with the maximum employment level being 4 times the average firm size. Sensitivity analysis indicated that 250 grid points for employment were enough to ensure that the discreteness of the grid does not affect the results. Finally, values implied by the perturbation solution were used as a starting point in the value function iteration.

Figures 7 and 8 show the demeaned simulated paths of the individual firm productivity cutoffs (exit decisions) and vacancy posting decisions for a given realization of the aggregate productivity shock.<sup>57</sup> The two are very similar. For simplicity, in both cases, firm employment is held fixed at its steady state level. Repeating the exercise for a different value of firm employment yields similar results. The figures show that the solution to the modified maximization problem follows the true one very well. The mean absolute percentage differences between the demeaned simulated paths based on the true and the modified maximization problem are 0.5% and 0.3% for the exit and vacancy posting decisions, respectively. The maximum absolute percentage differences are then 3.1% and 2.5% for the exit and vacancy posting decisions, respectively. These values occur during an extreme drop in aggregate productivity. Calculating the maximum absolute percentage differences without taking into account this extreme event gives 1.4% and 1.1% for the

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<sup>57</sup>For the value function iteration solution linear interpolation between grid points was used.

exit and vacancy posting decisions, respectively. These results suggest that the proposed procedure is a valid one for the problem at hand.

## C Non-stochastic cross-section simulation method

Instead of simulating a large number of firms characterized by their productivity and employment levels ( $p$  and  $n$ ), this method works with the *mass* of firms at grid points representing productivity and employment levels  $p$  and  $n$ . A first step is thus to create a fine grid for both individual firm productivity and employment levels. Denote the number of productivity grid points  $N_p$  and the number of employment grid points  $N_n$ .

Each grid point  $[p_i, n_j]$ , where  $i \in [1, 2, \dots, N_p]$  and  $j \in [1, 2, \dots, N_n]$ , thus gives the mass of firms with firm-specific productivity  $p_i$  and employment level  $n_j$ . The simulation can be described by the following iterative scheme

- 1. at each grid point firms decide whether or not to exit (based on past values of aggregate and firm-specific conditions).
- 2. new firms enter the economy based on the entry condition in (9) with employment  $n_0$  and a draw from the firm-specific productivity distribution  $G_0$ .
- 3. update aggregate productivity according to its law of motion in (5).
- 4. distribute all the mass at each grid point  $[p_i, n_j]$  to the grid points  $[p_k, n_j]$ , where  $k \in [1, 2, \dots, N_p]$ , according to the individual firm productivity law of motion in (6).
- 5. transfer all the mass at each grid point  $[p_i, n_j]$  to the grid point  $[p_i, n'_j]$ , where  $n'_j = (1 - \rho_x)n_j + v(p_i)q_t$ , with  $v(p_i)$  being the vacancy posting policy function. Go back to step 1.

The simulations in the current paper use 400 grid points for productivity and 400 grid points for employment. The grid points for (log) firm productivity run between  $-1.1$  to  $1.5$  and for the employment they are between  $0$  and  $100$ . Widening the ranges did not change the results. If the employment choice falls outside the grid range the respective mass is assigned to the appropriate corner grid point.

## D Implementation of government subsidy for young firms - details

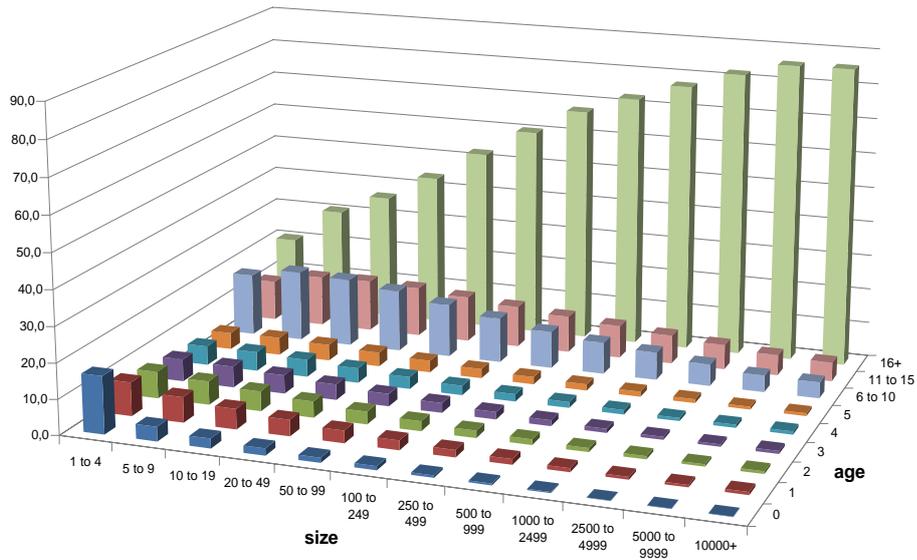
In this section I describe the technical implementation of the government subsidy to young firms. As mentioned in the main text, the government subsidy decays over time. This character of the subsidy is dictated by the solution method, since one needs to express it in recursive form. Let the initial subsidy a firm that has survived the first quarter be  $\tau_0$ . Then, the subsidy for older firms is assumed to be give by

$$\tau_j = \rho_\tau \tau_{j-1}, \quad (19)$$

where  $j$  indicates the age of a firm in quarters.  $\rho_\tau$  is determined such that firms of a given age (in the main text I consider 3 and 5 years) obtain 5% of  $\tau_0$ .

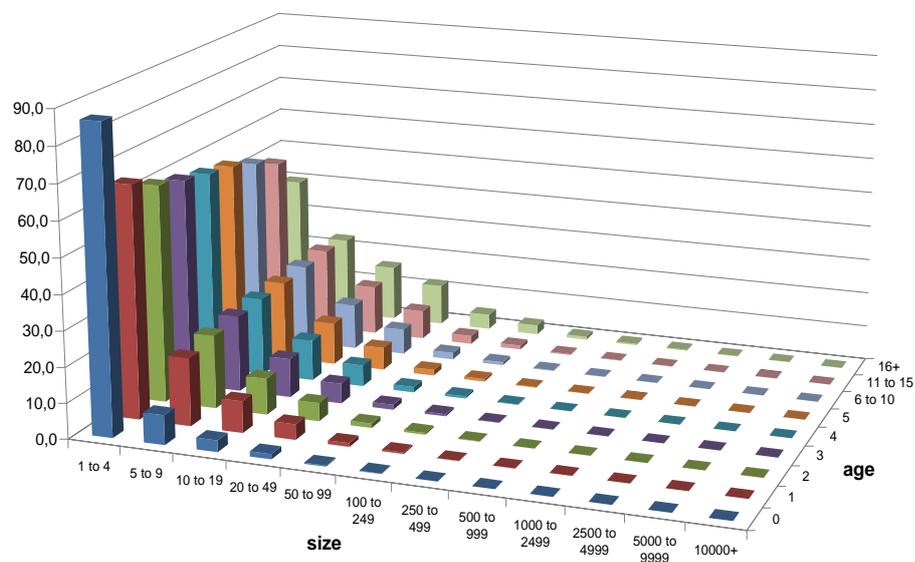
### Figures and tables

Figure 1: Firm age shares according to size



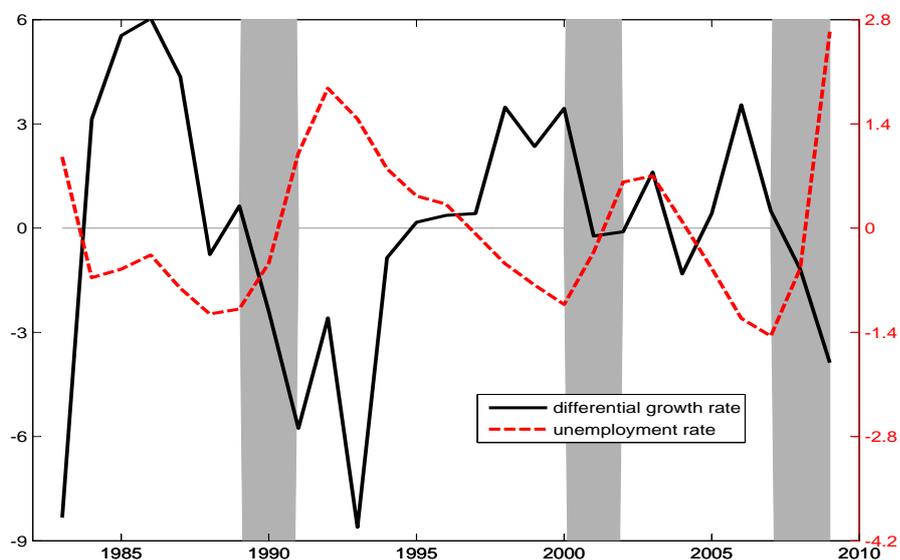
Notes: The figure plots the share of each firm age group in the total number of firms in a given size category. BDS data, averages between 1992 and 2009. The shares add up to 100 within each size dimension.

Figure 2: Firm size shares according to age



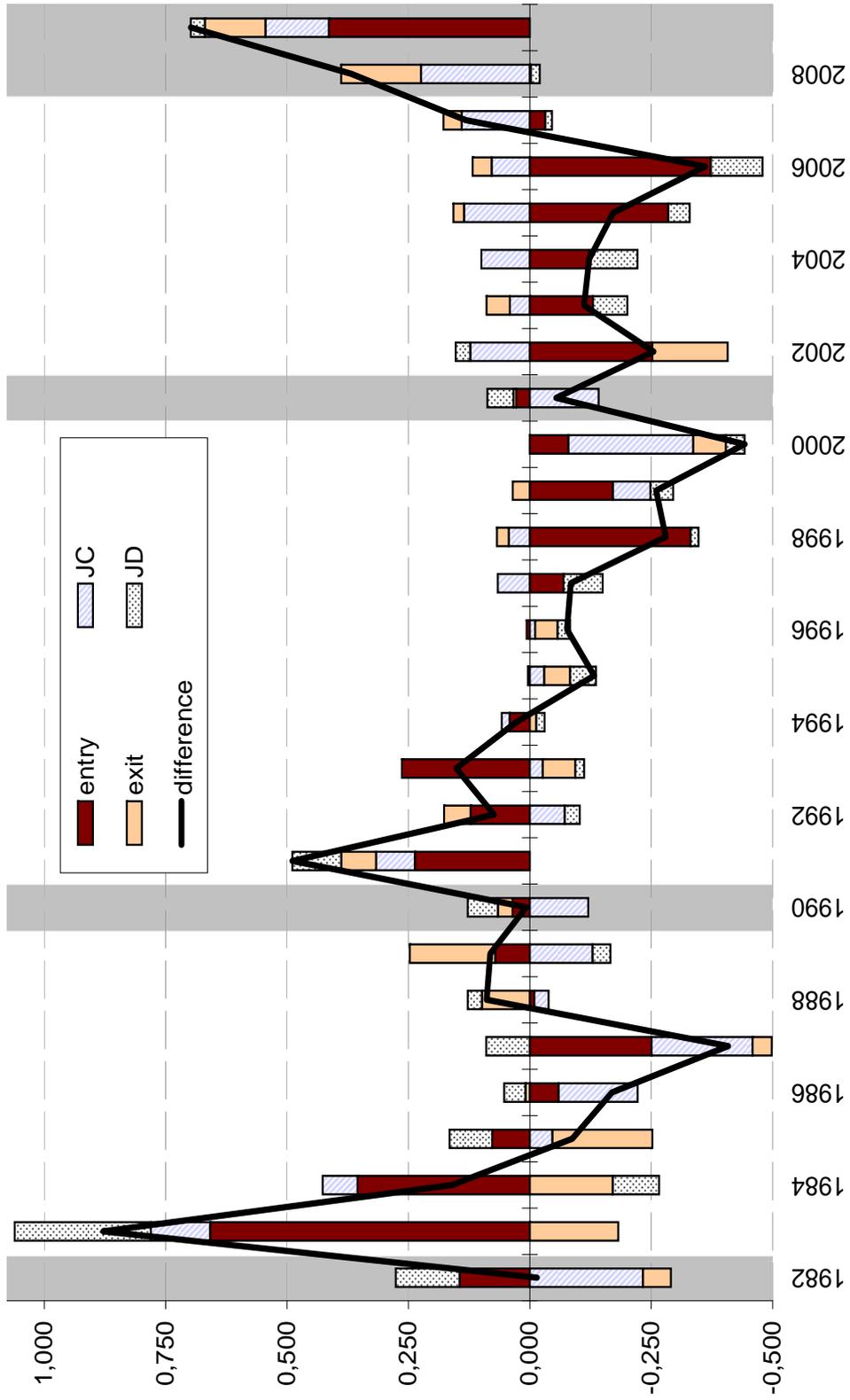
Notes: The figure plots the share of each firm size group in the total number of firms in a given age category. BDS data, averages between 1992 and 2009. The shares add up to 100 within each age dimension.

Figure 3: Differential employment growth rate and the unemployment rate



Notes: The figure plots the “differential growth rate” between young and old firm employment growth rates (left scale) and the “unemployment rate” (right scale), both detrended with a quadratic trend. The shaded areas are NBER recessions.

Figure 4: Actual-counterfactual unemployment decomposition; old firms



Notes: The figure plots the difference between the actual unemployment rate and the (mean adjusted) unemployment rate that arises when job creation and destruction rates of young firms are the same as those of old ("difference"). The bars indicate the contributions of new firms ("entry"), young firm closures ("exit"), job creation of young continuers ("JC") and job destruction of young continuers ("JD"). Shaded areas are NBER recessions.

Figure 5: Model time-line

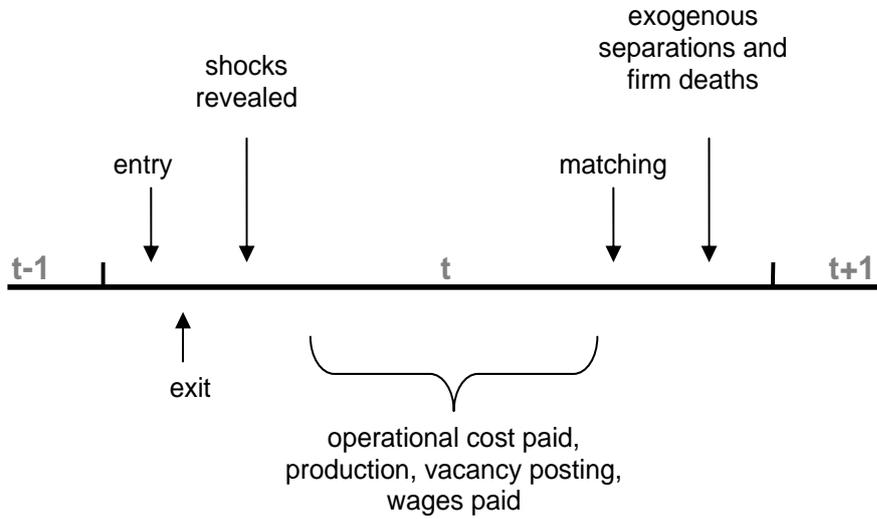
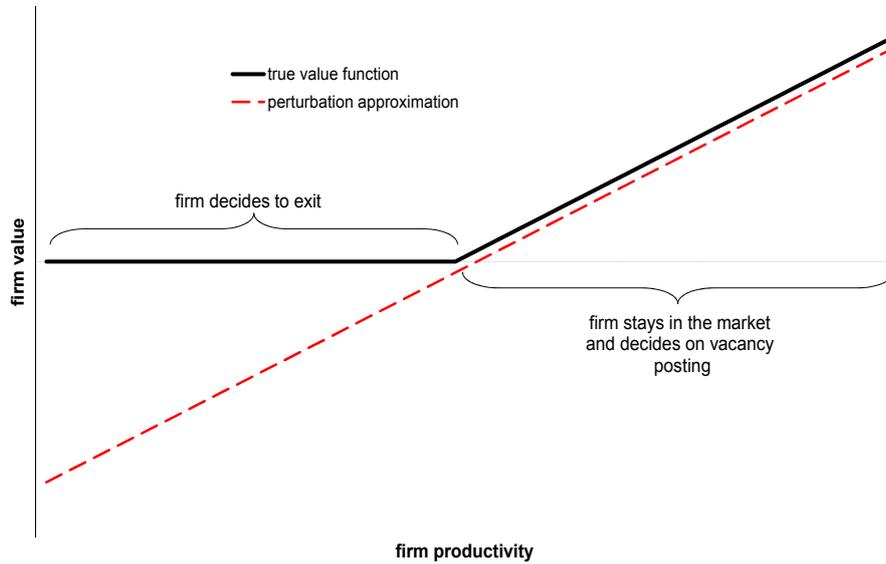
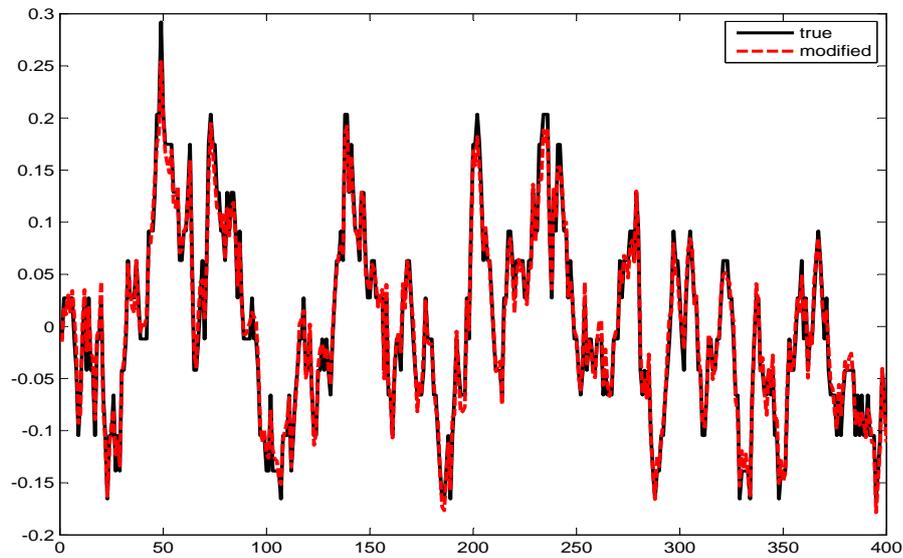


Figure 6: True value function and its perturbation approximation; illustration



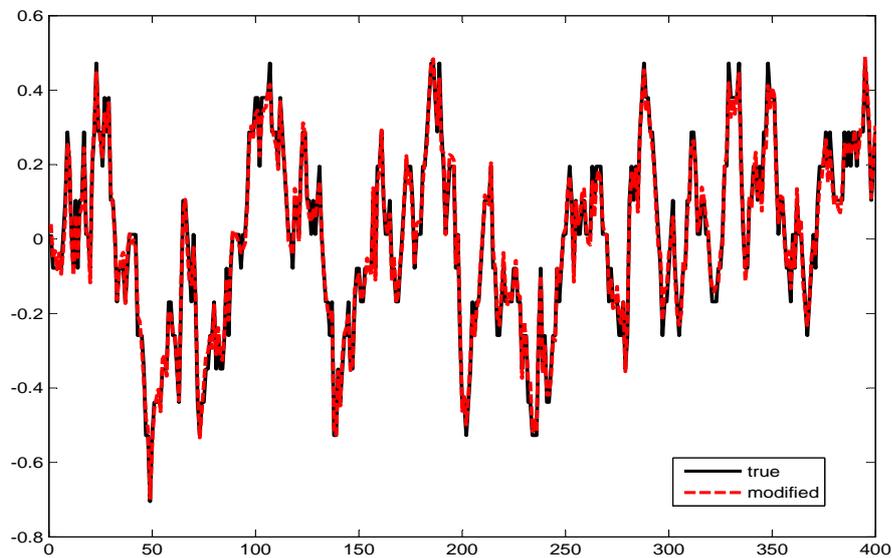
Notes: The figure shows a diagram of a fictional true value function that includes a non-differentiability together with its perturbation approximation that ignores it.

Figure 7: Comparing the solutions to the true and modified problem; exit decision



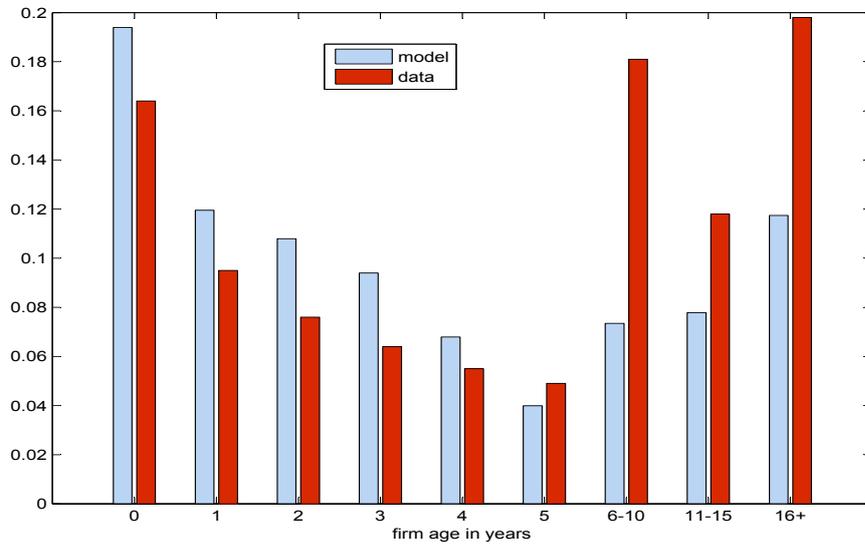
Notes: The figure compares individual firm exit decisions for a simulated series of aggregate productivity (keeping firm employment at its steady state) under the true and modified maximization problem..

Figure 8: Comparing the solutions to the true and modified problem; vacancy decision



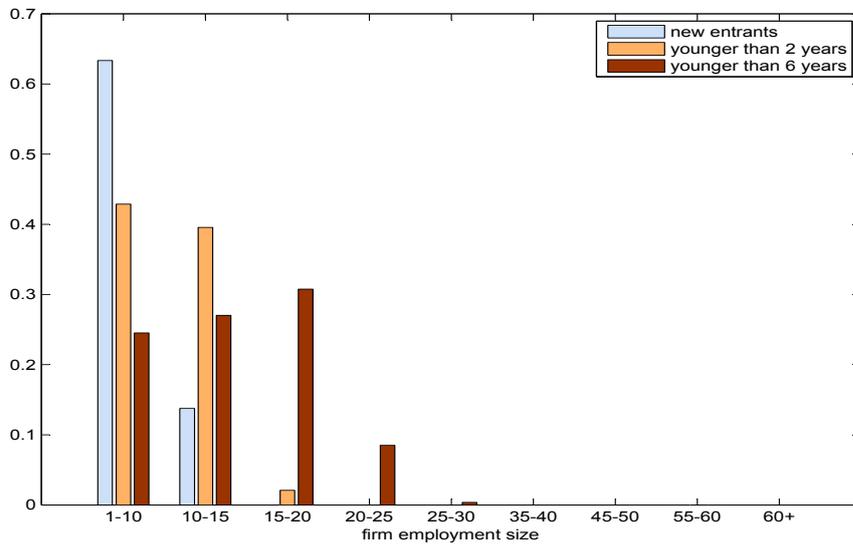
Notes: The figure compares individual firm vacancy posting decisions for a simulated series of aggregate productivity (keeping firm employment at its steady state) under the true and modified maximization problem.

Figure 9: Firm shares according to age for small firms, model and data



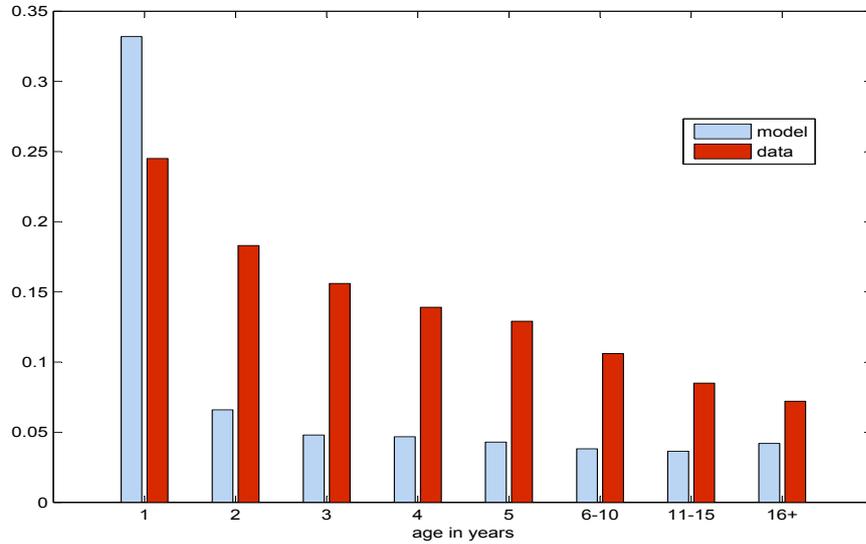
Notes: The figure plots the firm share of each firm age group in the total number of firms conditioning on small firms. Small firms are defined as the smallest 43%, which corresponds to firms with 1 to 4 employees in the data.

Figure 10: Firm shares according to size for young firms



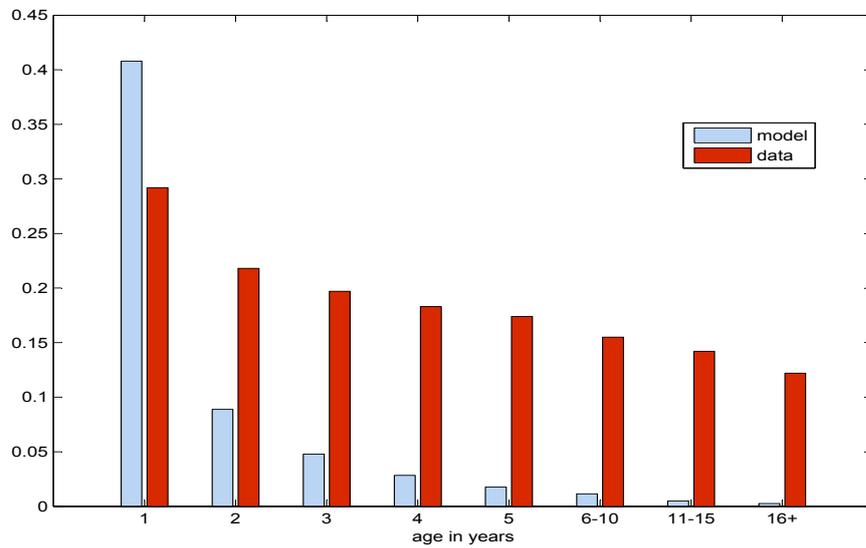
Notes: The figure plots the firm share of each firm size group in the total number of firms conditioning on young firms.

Figure 11: Firm exit rates according to age, model and data



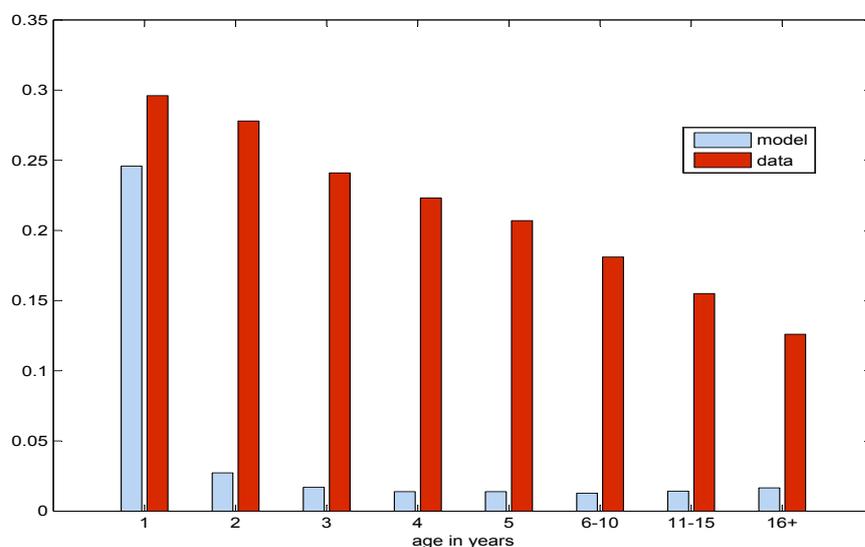
Notes: The figure plots firm exit rates of each firm age group. The exit rate is defined as the fraction of firms shutting down in the total number of firms in a given group.

Figure 12: Job creation rates according to age, model and data



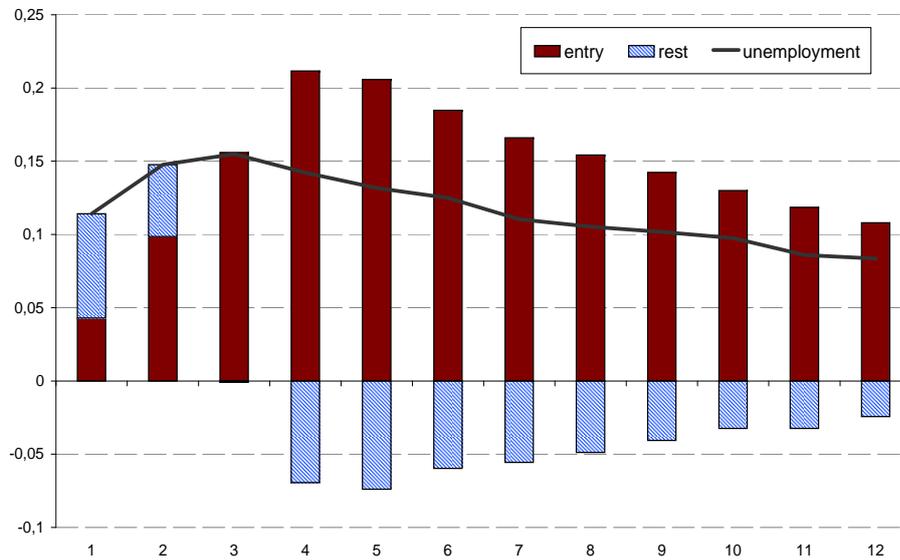
Notes: The figure plots job creation rates of each firm age group both for the mode and the data.

Figure 13: Job destruction rates according to age, model and data



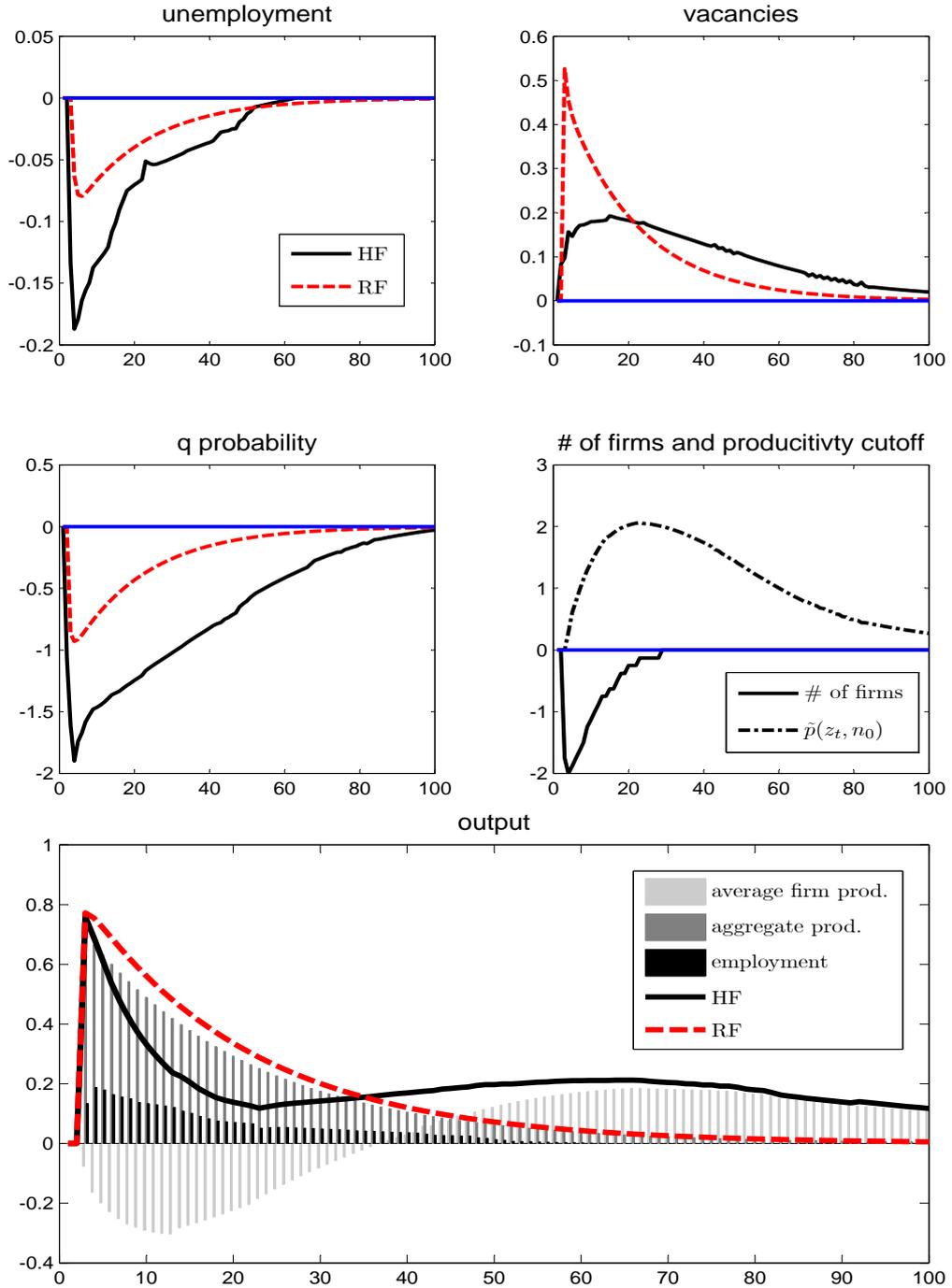
Notes: The figure plots job creation rates of each firm age group both for the mode and the data.

Figure 14: Decomposition of IRF of the unemployment rate



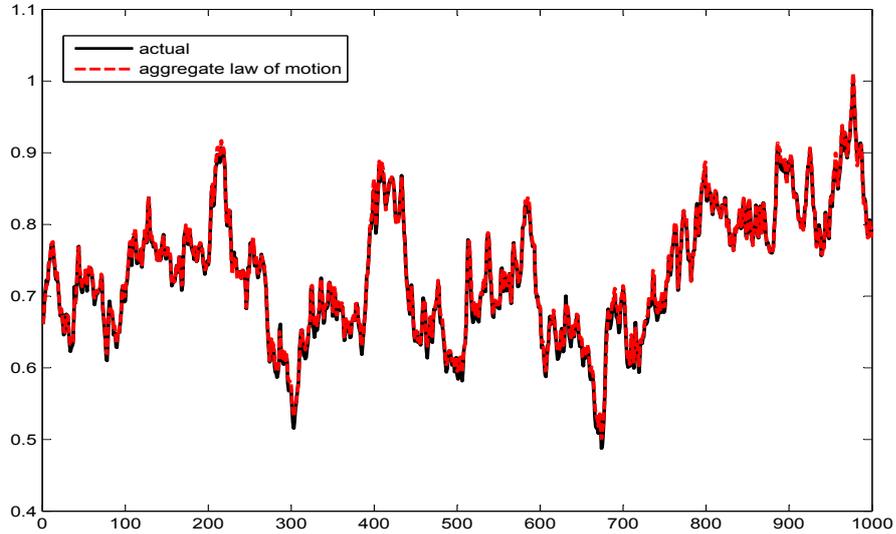
Notes: The figure plots the IRF of the unemployment rate to a negative one-standard-deviation shock to aggregate productivity. It further decomposes it into contribution of “entry” and a combined contribution of exit, job creation and destruction of continuing firms (“rest”).

Figure 15: IRFs to an aggregate productivity shock



Notes: The figure plots the IRFs of vacancies, unemployment, the probability of filling a vacancy and output generated by the heterogeneous firm (HF) model and a comparable representative firm (RF) version. The output response of the HF model is decomposed into contributions of the aggregate productivity shock, average firm productivity and employment. It further shows the IRFs of the number of all active firms and the cutoff value  $\tilde{p}(z_t, n_0)$  (these do not have counterparts in the RF model). “q probability” is the probability of filling a vacancy.

Figure 16: KS algorithm, accuracy plot for  $q$



Notes: The figure plots the simulated probability of filling a vacancy (“actual”) and the ones based on the aggregate law of motion used in by the firms in their maximization problems (“aggregate law of motion”). This series does *not* use at any point the true simulated value of  $q$ .

Table 1: Firm exit/entry rates and shares according to firm age, 1992-2009

age category	firm share	Exit	
		rate	share
0	10.3		
1	7.9	25.1	17.4
2	6.7	18.6	10.4
3	5.9	15.9	7.7
4	5.2	14.1	6.1
5	4.7	13.0	5.1
6 – 10	18.5	10.7	16.5
11 – 15	12.9	8.6	9.7
16+	28.0	7.0	27.1
all	1	10.5	1

Notes: “Firm share” gives the share of firms in a given category relative to the total number of firms. “Exit rate” is the fraction of firms that shut down in a given category relative to the number of firms in that age category. “Exit share” gives the fraction of firms shutting down in a given category relative to the total number of firms shutting down. Reported values are in percent.

Table 2: Job creation and destruction rates by firm age, 1992-2009

age category	net JC	gross JC	gross JD
0	200	100	0
1	-2.2	28.0	30.2
2	-6.5	20.9	27.5
3	-5.1	18.9	24.0
4	-4.3	17.8	22.1
5	-3.7	16.9	20.6
6 – 10	-3.0	15.0	17.9
11 – 15	-1.9	13.7	15.6
16+	-0.5	12.0	12.5
all	1.5	16.2	14.8

Notes: Net and gross job creation rates and gross job destruction rates for different firm age groups.

Table 3: Job creation and destruction shares by firm age, 1992-2009

age category	gross JC	gross JD	employment share
0	17.4	0	1.4
1	4.9	5.8	2.8
2	3.4	5.0	2.7
3	2.9	4.0	2.5
4	2.6	3.5	2.4
5	2.4	3.2	2.3
6 – 10	9.5	12.6	10.3
11 – 15	7.7	9.6	9.1
16+	49.2	56.3	66.5

Notes: Shares of gross job creation, destruction and employment of different firm age groups in the total.

Table 4: Unemployment rate decomposition; 2008-2009

	2008	2009
unemployment	5.81	9.29
unemployment <sub>count</sub>	5.48	8.24
difference	0.33	1.05
- entry	0.26	0.60
- JC	0.13	0.26
- exit	0.04	-0.00
- JD	-0.10	0.19

Notes: The first two lines show the actual and the counterfactual unemployment rate based on young firms behaving as they would have based on their average response during the previous recessions. The bottom rows show the difference between the two and its decomposition into entry, exit, job creation and destruction of continuing firms.

Table 5: Model parameters

Parameter		Value	Target/Source
Discount factor	$\beta$	0.99	
Autocorr. coef., agg. shock	$\rho_z$	0.95	
St. dev, agg. shock	$\sigma_z$	0.007	
Exog. separation rate	$\rho_x$	0.828	0.828 $\rho_{total}$ , Helfand et. al. (2007)
Exog. exit probability	$\delta$	0.0014	$\delta = 0.017\rho_x/0.983$ , Helfand et. al. (2007)
Match elasticity	$\mu$	0.72	Shimer (2007)
Home production	$b$	0	normalization
Match efficiency	$\gamma$	0.741	$u = 0.12$
Operational cost	$\xi$	0.562	$\rho_{total} = 0.1$
Scale vacancy cost	$\kappa_0$	0.159	$q = 0.71$ , Den Haan et. al. 2000
Scale entry prob.	$\alpha_0$	6.1e3	entry = exit in steady state
Entry cost	$\Psi$	1	normalization
Power entry prob. par.	$\alpha_1$	-2.92	$\frac{\sigma((n_{entry})/n)}{\sigma(\log y/n)} = 0.24$
1 <sup>st</sup> wage parameter	$\tilde{\omega}$	0.943	$\frac{\sigma(n/L)}{\sigma(\log y/n)} = 0.53$
2 <sup>nd</sup> wage parameter	$\omega$	0.25	$\frac{\sigma(\log w)}{\sigma(\log y/n)} = 0.76$
Idio. shock st. dev.	$\sigma_p$	0.073	$\sigma(g) = 0.56$ , Davis et. al. 2010
Idio. shock persistence	$\rho_p$	0.976	new job persistence 0.68, Davis et.al. 1996
Initial idio. shock st. dev.	$\sigma_{p,0}$	0.251	average firm size 21.3
Initial employment	$n_0$	5.793	average size of new firms 6.1

Notes:  $\rho_{total}$  is the total separation rate,  $n_{entry}$  is total employment of new firms,  $\sigma(g)$  is the dispersion of employment growth rates (including entry and exit) and new job persistence is defined as according the Davis, Haltiwanger, and Schuh (1996) as the fraction of new jobs that survive into the next quarter.

Table 6: Results of firm growth regressions

Regressor	Age only	Size only	Age and size
<i>Regressions using dummy variables for age and size groups</i>			
<i>Age groups</i>			
0	2.000 (0)		2.552 (0.0015)
1	0.026 (0.0008)		0.046 (0.0014)
2	0.009 (0.0007)		0.127 (0.0013)
3	-0.004 (0.0006)		0.125 (0.0011)
4	-0.013 (0.0006)		0.089 (0.0009)
5	-0.018 (0.0003)		0.050 (0.0006)
6-10	-0.026 (0.0003)		0.004 (0.0003)
11-15	-0.030 (0.0002)		-0.014 (0.0003)
16+	-0.031 (0.0001)		-0.016 (0.0002)
<i>Size groups</i>			
0-8		0.194 (0.0007)	-0.552 (0.0015)
8-12		0.041 (0.0008)	0.007 (0.0010)
12-16		0.015 (0.0011)	-0.061 (0.0019)
16-20		-0.001 (0.0010)	-0.123 (0.0017)
20-24		-0.065 (0.0007)	-0.149 (0.0010)
24-30		-0.061 (0.0009)	-0.068 (0.0011)
30-36		-0.009 (0.0002)	-0.005 (0.0003)
36-42		-0.003 (0.0001)	0.006 (0.0002)
42+		-0.003 (0.0001)	
<i>Regressions using age and size, not grouped</i>			
Age	-0.0008 [-15.18] (0.5e - 5) (0.052)		-0.0007 [-10.89] (0.5e - 5) (0.054)
Size		-0.0041 [-10.81] (1.9e - 5) (0.053)	-0.0010 [-2.18] (2.4e - 5) (0.052)

Notes: The model is simulated 1,000 times, each simulation has 1,108 time periods and the first 1,000 are dropped to obtain 108 observations as in the empirical counterpart. “Regressor” indicates either a dummy variable for a certain age/size category, or actual age and size values. The table reports coefficients of size and age in a regression on firm employment growth averaged across the 1,000 simulations. The square brackets report the respective t-statistics averaged across simulations. The round brackets give the standard deviations of the given coefficient or t-statistic across simulations.

Table 7: Standard deviations and cross-correlations of selected variables; U.S. data

	u	v	v/u	n	f	y
standard deviation	0.54	0.37	12.25	0.53	3.18	1.14
autocorrelation	0.87	0.89	0.92	0.87	0.81	0.84
correlation matrix						
u	1					
v	-0.85	1				
v/u	-0.88	0.93	1			
n	-0.99	0.85	0.88	1		
f	-0.88	0.77	0.97	0.88	1	
y	-0.81	0.82	0.81	0.81	0.74	1

Notes: The data used are the following: the unemployment rate is taken from the BLS, the vacancy rate is taken from Barnichon (2010), the employment to labor force ratio (“n”) is taken from the BLS, “f” is the job finding rate taken from Shimer (2007) and “y” is real GDP published by the BEA. The data are quarterly and run from 1982Q1 to 2007Q1 (the end of the sample is dictated by availability of the job finding rate data). Real GDP is logged and all variables are detrended with an HP filter (smoothing coefficient 1,600). The reported standard deviations are relative to the standard deviation of labor productivity (output per worker in the non-farm business sector).

Table 8: Standard deviations and cross-correlations of selected variables; HF model

	u	v	v/u	n	f	y
standard deviation	0.54	0.30	6.87	0.53	1.39	1.11
	(0.021)	(0.011)	(0.37)	(0.024)	(0.056)	(0.041)
autocorrelation	0.88	0.72	0.82	0.88	0.83	0.79
	(0.014)	(0.012)	(0.021)	(0.014)	(0.022)	(0.017)
correlation matrix						
u	1					
v	-0.55	1				
	(0.039)					
v/u	-0.94	0.79	1			
	(0.038)	(0.015)				
n	-1.00	0.55	0.94	1		
	(0.00)	(0.079)	(0.039)			
f	-0.95	0.79	1.00	0.95	1	
	(0.039)	(0.015)	(0.00)	(0.039)		
y	-0.91	0.61	0.89	0.91	0.90	1
	(0.033)	(0.027)	(0.021)	(0.033)	(0.021)	

Notes: The heterogeneous firm model was simulated 1,000 times, each simulation lasted for 1,108 quarters where the first 1,000 were dropped to obtain a sample of 27 years as in the data. The reported statistics are averages over the 1,000 simulations (standard deviations across simulations are in brackets). The data definitions and the treatment of the simulated data are the same as in Table 7.

Table 9: Standard deviations and cross-correlations of selected variables; RF model

	u	v	v/u	n	f	y
standard deviation	0.54 (0.014)	1.82 (0.054)	18.44 (0.520)	0.53 (0.014)	4.34 (0.165)	1.09 (0.015)
autocorrelation	0.78 (0.050)	0.60 (0.082)	0.74 (0.062)	0.78 (0.050)	0.73 (0.062)	0.72 (0.064)
correlation matrix						
u	1					
v	-0.56 (0.086)	1				
v/u	-0.73 (0.063)	0.97 (0.006)	1			
n	-1.00 (0.00)	0.56 (0.086)	0.73 (0.063)	1		
f	-0.73 (0.063)	0.98 (0.005)	0.99 (0.002)	0.73 (0.063)	1	
y	-0.71 (0.065)	0.98 (0.002)	0.99 (0.001)	0.71 (0.065)	0.99 (0.001)	1

Notes: the reported business cycle statistics were generated by the representative firm model. For further details see note of Table 8.

Table 10: Results of subsidizing young firms

variable/subsidy for	Entrants	Existing young firms	
		3 years of age	5 years of age
unemployment	99.2	99.5	98.9
output	100.6	98.5	98.1
number of firms	106.7	103.1	104.8
number of new firms	119.4	94.5	91.4
average firm productivity	101.2	99.1	98.4
average exit rate	111.9	91.6	87.1
probability q	96.4	98.5	97.7

Notes: In all cases the total costs of subsidies are 1% of aggregate steady state output. Subsidy for entrants means that only firm entry is supported and existing firms (of all ages) are not subsidized. Subsidy for existing young firms of 3 and 5 years of age means that firms with 2-12 and 2-20 quarters of age, respectively, are subsidized such that the subsidy monotonically falls with age (see Appendix D for details). The table shows steady state values of variables relative to those in the case of no subsidy. In the case of output, the results are *net* of the costs of the subsidy.