Solution Methods for State-dependent and Time-dependent Models

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Summary: Dynamic stochastic economic models are normally built on the assumption of stationary environment, namely, it is assumed that the economy's fundamentals such as preferences, technologies and laws of motions for exogenous variables do not change over time. Such models have stationary solutions in which optimal value and decision functions depend on the current state but not on time. The state-dependent class of models is convenient for applied work since time-invariant solutions are relatively easy to construct.

At the same time, real-world economies constantly evolve over time, experiencing population growth, technological progress, trends in tastes and habits, policy regime changes, evolution of social and political institutions, etc. Also, economic policies change over time, for example, Central Banks can change parameters in the Taylor rule or employ time-dependent unconventional monetary policies such as forward guidance. Modeling time-dependent features of the data would require us to assume that some parameters of economic models change over time, following deterministic and or stochastic trends. The resulting models are nonstationary, and their optimal value and decision functions are time-dependent. Conventional numerical methods used for stationary infinite-horizon models are not suitable for analyzing such nonstationary models.

In the workshop, we first review numerical techniques for state-dependent models by focusing on problems with a large number of state variables, including Smolyak method, cluster and epsilon-distinguishable set grids, monomial formulas, precomputation techniques, as well as endogenous grid and envelope condition methods. (MATLAB codes with documentation will be provided for all these methods). We illustrate the applications of these methods by using examples of one and multi-agent neoclassical growth models, as well as a large-scale new Keynesian model.

We then show a quantitative framework, called extended function path (EFP), for calibrating, solving, simulating and estimating time-dependent models. We apply EFP to solve a collection of challenging nonstationary time-dependent and unbalanced-growth applications, including stochastic growth models with parameters shifts and drifts, capital augmenting technological progress, anticipated regime switches, time-trends in volatility of shocks, seasonal fluctuations as well as new Keynesian economies with time-varying parameters. Also, we show an example of estimation and calibration of parameters in an unbalanced growth model using the data on the U.S. economy. Again, MATLAB codes for solving time-dependent models will be provided to the workshop participants.

The papers:

For general background on global solution methods for large-scale models, we will use:

i) Lilia Maliar and Serguei Maliar, (2014). "Numerical Methods for Large Scale Dynamic Economic Models", in: Schmedders, K. and K.L. Judd (Eds.), **Handbook of Computational Economics, Volume 3, Chapter 7, 325-477, Amsterdam: Elsevier Science**.

Other papers on state-dependent large-scale models that we will cover are:

ii) Kenneth L. Judd, Lilia Maliar, Serguei Maliar and Rafael Valero, (2014). "Smolyak Method for Solving Dynamic Economic Models: Lagrange Interpolation, Anisotropic Grid and Adaptive Domain", Journal of Economic Dynamic and Control 44(C), 92-123.

iii) Lilia Maliar and Serguei Maliar, (2015). "Merging Simulation and Projection Approaches to Solve High-Dimensional Problems with an Application to a New Keynesian model", Quantitative Economics 6, 1-47.

iv) Kenneth L. Judd, Lilia Maliar, Serguei Maliar and Inna Tsener, (2016). "How to Solve Dynamic Stochastic Models Computing Expectations Just Once"", Quantitative Economics (forthcoming).

v) Cristina Arellano, Lilia Maliar, Serguei Maliar and Viktor Tsyrennikov, (2016). "Envelope Condition Method with an Application to Default Risk Models", Journal of Economic Dynamics and Control 69, 436-459.

vi) Kenneth L. Judd, Lilia Maliar and Serguei Maliar, (2016). "Lower Bounds on Approximation Errors to Numerical Solutions of Dynamic Economic Models", Econometrica (forthcoming).

vii) Vadym Lepetuyk, Lilia Maliar and Serguei Maliar (2016). "Should Central Banks Worry about Nonlinearities of Their Large-Scale Macroeconomic Models?"

Time-dependent models are analyzed by using the EFP framework developed in:

viii) Lilia Maliar, Serguei Maliar, John B. Taylor and Inna Tsener (2015). "A Tractable Framework for Analyzing a Class of Nonstationary Markov Models", **NBER 21155**.