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# A Defense of Moderation in Monetary Policy

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**Abstract**

This paper examines the implications of uncertainty about the effects of monetary policy for optimal monetary policy with an application to the current situation. Using a stylized macroeconomic model, I derive optimal policies under uncertainty for both conventional and unconventional monetary policies. According to an estimated version of this model, the U.S. economy is currently suffering from a large and persistent adverse demand shock. Optimal monetary policy absent uncertainty would quickly restore real GDP close to its potential level and allow the inflation rate to rise temporarily above the longer-run target. By contrast, the optimal policy under uncertainty is more muted in its response. As a result, output and inflation return to target levels only gradually. This analysis highlights three important insights for monetary policy under uncertainty. First, even in the presence of considerable uncertainty about the effects of monetary policy, the optimal policy nevertheless responds strongly to shocks: uncertainty does not imply inaction. Second, one cannot simply look at point forecasts and judge whether policy is optimal. Indeed, once one recognizes uncertainty, some moderation in monetary policy may well be optimal. Third, in the context of multiple policy instruments, the optimal strategy is to rely on the instrument associated with the least uncertainty and use alternative, more uncertain instruments only when the least uncertain instrument is employed to its fullest extent possible.

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# 1 Introduction

The Federal Reserve has been criticized by some for not acting aggressively enough to meet its statutory mandate of maximum employment and price stability (see, for example, Krugman, 2012). Inflation is below the Federal Reserve's preferred 2 percent rate and the unemployment rate is elevated and, according to many forecasts, these conditions are expected to persist for some time. These commentators argue that a more expansionary monetary policy stance would lower the path of the unemployment rate and raise the inflation rate. According to this argument, under reasonable assumptions regarding preferences over the two objectives, such a policy would bring the Fed closer to its mandated objectives more quickly.

The claim that the Fed is responding insufficiently to the shocks hitting the economy rests on the assumption that policy is made with complete certainty about the effects of policy on the economy. Nothing could be further from the truth. Policymakers are unsure of the future course of the economy and uncertain about the effects of their policy actions. Uncertainty about the effects of policies is especially acute in the case of unconventional policy instruments such as using the Fed's balance sheet to influence financial and economic conditions. And, as is well known since the seminal work of Brainard (1967), uncertainty about the effects of policy may be an argument for attenuated policy responses to shocks. This conservatism principle of optimal policy under uncertainty implies that one cannot necessarily infer from point forecasts of persistently low inflation and elevated unemployment that monetary policy is suboptimal.

This paper examines the implications of uncertainty for optimal monetary policy with an application to the current situation. I start with a stylized static macroeconomic model and derive the optimal monetary policy under both certainty and uncertainty. I then examine optimal policy in the current environment using an estimated version of the stylized model. In this model, the Federal Reserve possesses two monetary policy instruments: the federal funds rate and unconventional balance sheet policies. The federal funds rate is assumed to be subject to a zero lower bound, giving rise to the use of the balance sheet instrument. Based on empirical studies, uncertainty regarding the effects of unconventional policies on the economy is greater than for conventional policies.

According to the estimated stylized model, the U.S. economy is currently suffering from a large and persistent adverse demand shock. Optimal monetary policy absent uncertainty would quickly restore real GDP close to its potential level and allow the inflation rate to temporarily rise somewhat above the 2 percent longer-run target. Thus, this model captures the argument of critics that more aggressive policy could bring the economy rapidly close to target levels in a world of perfect certainty. In contrast, the optimal policy under uncertainty is much more muted in its response to the negative demand shock. As a result, output and inflation return to their target levels only gradually.

This analysis highlights three important insights for monetary policy under uncertainty. First, even in the presence of considerable uncertainty about the effects of monetary policy, the optimal policy nevertheless responds strongly to shocks: uncertainty does not imply inaction. Indeed, in the estimated model, the optimal conventional and unconventional policy responses in the current situation are quite strong, just not as strong as would be called for absent uncertainty. Second, one cannot simply look at point forecasts and judge whether policy is optimal or not. One needs to evaluate policy in the context of the distribution of forecasts that accounts for uncertainty. Indeed, once one recognizes uncertainty, some moderation in monetary policy may well be optimal. Third, in the context of multiple policy instruments, the optimal strategy is to rely on the instrument with the least uncertainty and use other, more uncertain instruments, only when the least uncertain instrument is employed to its fullest extent possible.

The paper is organized as follows. Section 2 describes a simple static model and derives optimal monetary policy under certainty and uncertainty. Section 3 extends the model to include an unconventional monetary policy instrument and the zero lower bound on nominal interest rates. Section 4 reports optimal monetary policy based on empirical estimates of uncertainty regarding the effects of monetary policy actions. Section 5 applies these results to the current situation and reports forecasts under alternative monetary policy assumptions. Section 6 examines the effects of costs to unconventional monetary policy not captured by the model. Section 7 concludes.

## 2 Optimal Conventional Monetary Policy in a Simple Static Model

This section analyzes a simple static macroeconomic model that facilitates the analytical derivation of the optimal conventional monetary policy, both under certainty and uncertainty. By conventional monetary policy, I mean the use of the short-term interest rate as the policy instrument. This stylized model exactly corresponds to the textbook New Keynesian model in the case of i.i.d. shocks, the absence of a commitment technology on the part of the policymaker, and a zero target rate of inflation (Levin and Williams 2003). In the following section, the model is extended to incorporate unconventional monetary policy and the zero lower bound.

The central bank seeks to minimize expected squared fluctuations in the inflation gap (the difference between the inflation rate and the central bank's target rate of inflation) and the output gap (defined as the percent deviation between the actual level of output and potential output). Specifically, the central bank's objective is to minimize the quadratic loss:

$$\mathcal{L} = E\{(\pi - \pi^*)^2 + \lambda y^2\}, \tag{1}$$

where $\pi$ denotes the inflation rate, $\pi^*$ is the target rate of inflation, $y$ is the output gap, and $E$ denotes expectations. The parameter $\lambda \geq 0$ is the fixed weight the policymaker places on output gap stabilization relative to inflation stabilization. The target inflation rate is assumed to be constant.

The model economy is described by equations for the output gap and the inflation rate. The output gap equation is given by:

$$y = -\eta(r - r^*) + u, \tag{2}$$

where $r$ denotes the short-term nominal interest rate, $r^*$ is the constant equilibrium interest rate, and $u$ is an i.i.d. mean zero demand shock with variance $\sigma_u^2$. Note that the output gap is assumed to be negatively related to the nominal interest rate, not the real interest rate. This assumption simplifies the derivations, but does not materially affect the main results of the paper. The inflation equation is given by:

$$\pi = \pi^* + \kappa y + e, \tag{3}$$

where $e$ is an i.i.d. mean zero inflation shock with variance $\sigma_e^2$. Note that if the output gap and the inflation shock are both zero, the inflation rate equals the target rate of inflation.

## 2.1 Optimal Policy Absent Parameter Uncertainty

I start by deriving the optimal policy assuming that all features of the model including the model parameters $\eta$ and $\kappa$ are known with certainty by the policymaker. In the following, the resulting optimal policy is referred to as the certainty equivalent policy. The policymaker is assumed to observe the realizations of the shocks when the value of the policy instrument is chosen. Therefore, there is no uncertainty at the time the policy decision is made. Given the linear-quadratic specification of the model, the optimal policy can be described as a linear reaction function in terms of the equilibrium interest rate and shocks:

$$r = r^* + \gamma u + \phi e, \tag{4}$$

where $\gamma$ and $\phi$ are coefficients chosen by the policymaker. After substitution of the policy rule, the expected loss is given by:

$$\mathcal{L} = E\left\{\lambda\Big((1-\eta\gamma)u - \eta\phi e\Big)^2 + \Big(\kappa(1-\eta\gamma)u + (1-\kappa\eta\phi)e\Big)^2\right\}, \tag{5}$$

where expectations are taken with respect to the two shocks. The parameters describing the certainty equivalent optimal policy (denoted with a superscript C to denote certainty) are given by:

$$\gamma^C = \frac{1}{\eta}, \tag{6}$$

$$\phi^C = \frac{\kappa}{(\lambda+\kappa^2)\eta}. \tag{7}$$

The optimal policy absent parameter uncertainty displays two basic principles of monetary policy in models such as this. First, it completely offsets the effects of a demand shock, regardless of the central bank's weight on output gap stabilization in its objective function. The demand shock in this model creates a "divine coincidence" of goals, where output gap and inflation stabilization are perfectly aligned and there is no trade-off between the two objectives. Second, the inflation shock creates a short-run trade-off between the two goals, and the optimal policy response depends on the degree of concern for output gap stabilization. In the limiting case of $\lambda = 0$, where the policymaker cares only about inflation, the optimal policy creates an output gap that completely offsets the shock's effect on inflation. For $\lambda > 0$, the optimal policy partially offsets the effect of the shock on inflation by moving output in the opposite direction. This response reflects the trade-off between the inflation and output gap objectives inherent to an inflation shock. The greater the value of $\lambda$, the more muted the response to the inflation shock.

## 2.2 Optimal Policy with Parameter Uncertainty

I now turn to the problem of optimal policy when the policymaker is uncertain about key parameters of the model economy, a problem originally studied by Brainard (1967). To keep things analytically tractable, I assume that the policymaker's uncertainty is limited to the values of two model parameters related to the monetary transmission channel, $\eta$ and $\kappa$. (Uncertainty about $r^*$ has no effect on optimal policy in this simple model, so I ignore it in this paper.)

The policymaker is assumed to follow a Bayesian approach with priors over the means and variances of these two parameters. In addition, the distributions of the two parameters are assumed to be independent of each other and independent of the shocks to the output gap and inflation equations. As before, the policymaker chooses the value of the policy instrument to minimize the expected loss, where expectations are taken with respect to the shocks and the two unknown parameters.[1] Note that this approach treats the uncertainty as a static one-time problem, and I abstract from learning about the model parameters (see Wieland 2000 for analysis of optimal experimentation in the context of parameter uncertainty and learning).

The optimal policy with parameter uncertainty is described by the same specification of linear policy reaction function as before. After taking expectations of the shocks, the first-order condition for the optimal response to the demand shock ($\gamma$) is given by:

$$E\left\{(\lambda+\kappa^2)\eta(1-\eta\gamma)\sigma_u^2\right\}=0. \tag{8}$$

The first-order condition for the optimal response to the inflation shock ($\phi$) is given by:

$$E\left\{\Big((\lambda+\kappa^2)\eta^2\phi-\kappa\eta\Big)\sigma_e^2\right\}=0. \tag{9}$$

Taking expectations yields the solution for the optimal values of the policy parameters, denoted with a superscript B (for Bayesian):

$$\gamma^B=\frac{\bar{\eta}}{\bar{\eta}^2+\sigma_\eta^2}, \tag{10}$$

[1] The assumption that the policymaker observes the shocks is made for simplicity. In an environment of parameter uncertainty, the shocks themselves may be measured with error, as discussed in Edge et al. (2010).

$$\phi^B = \frac{\bar{\eta}\bar{\kappa}}{(\lambda + \bar{\kappa}^2 + \sigma_\kappa^2)(\bar{\eta}^2 + \sigma_\eta^2)}, \tag{11}$$

where the prior mean of parameter $x$ is denoted by $\bar{x}$ and its prior variance its donated by $\sigma_x^2$.

The optimal responses to the two shocks are muted relative to the case of no parameter uncertainty. The degree of attenuation of the optimal response to the demand shock depends only on the precision of one's estimate of the interest rate sensitivity of the output gap, $\eta$. In the case of inflation shocks, the optimal degree of attenuation depends on the uncertainty regarding both $\eta$ and $\kappa$, as well as $\bar{\kappa}$ and $\lambda$. The intuition for policy attenuation is that uncertainty about the effects of policy creates ex post policy errors that cause economic outcomes to differ from the policymaker's intentions. The magnitude of the policy error is multiplicative in the policy action; that is, the larger the action, the greater the expected squared error. Therefore, the expected size of the policy error is affected by the size of the policy action, creating a bias toward muted policy actions.

It is convenient to express the degree of attenuation of the optimal policy response owing to parameter uncertainty in terms of the precision of the parameter estimates as measured by the t-statistics implied by the policymaker's prior. Let $t_\eta$ denote the ratio of the prior mean of this parameter and its prior standard deviation, $t_\eta \equiv \bar{\eta}/\sigma_\eta$. This corresponds to a standard t-statistic for the parameter $\eta$. The optimal policy response to demand shocks under parameter uncertainty is proportional to that under certainty, with the factor of proportionality depending on $t_\eta$:

$$\gamma^B = \gamma^C \frac{1}{1 + t_\eta^{-2}}. \tag{12}$$

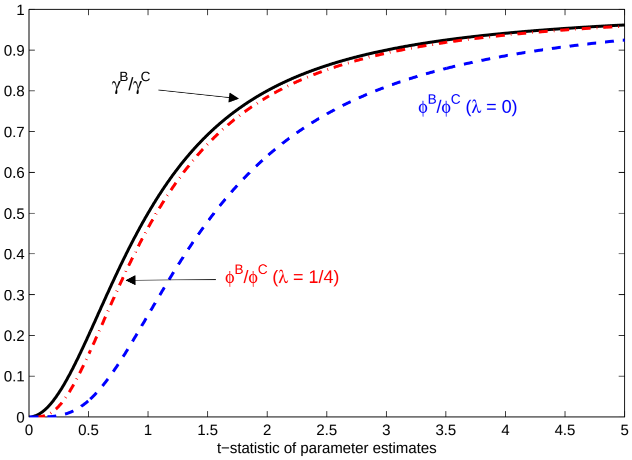
This relationship is shown by the black solid line in Figure 1 for a range of values of $t_\eta$ given on the x-axis. For example, if the t-statistic associated with $\eta$ is 1, then the optimal policy response to a demand shock is 50 percent as large as when uncertainty is absent, and if the t-statistic is 2, the optimal policy response is 80 percent as large.

The optimal value of $\phi^B$ is proportional to that assuming no parameter uncertainty, with the factor of proportionality given by:

$$\phi^B = \phi^C \frac{1}{\left(1 + t_\eta^{-2}\right)\left(1 + \frac{\bar{\kappa}^2}{\lambda + \bar{\kappa}^2} t_\kappa^{-2}\right)}, \tag{13}$$

where $t_\kappa$ denotes the ratio of $\bar{\kappa}$ divided by the standard deviation of the prior. Note that, as in the case of the optimal response to demand shocks, the precision of the estimate of $\eta$ is a key

**Figure 1: Optimal Policy under Uncertainty**




Notes: The black solid line shows the ratio of the optimal policy response to a demand shock under uncertainty to the optimal policy absent uncertainty as a function of the precision of the estimates of model parameters. The blue dashed and red dash-dot lines show the ratios of the optimal policy response to an inflation shock under uncertainty to the optimal policy absent uncertainty, for values of $\lambda$ of 0 and 0.25, respectively.

determinant in the degree of attenuation in response to inflation shocks. Figure 2 displays the degree of attenuation as a function of the imprecision of the estimates of $\eta$ and $\kappa$ for two values of $\lambda$. To construct this figure, I assume $\kappa = 0.145$, based on the estimation results reported below. The blue dashed line shows the results for $\lambda = 0$; the red dash-dot line shows the results for $\lambda = 1/4$. Results for larger values of $\lambda$ are not shown because they are virtually indistinguishable from the case of the optimal response to demand shocks, shown by the $\gamma^B/\gamma^C$ line in the figure. In this figure, the precision of the estimates of the two parameters, as measured by the t-statistic, is assumed to be the same and is indicated on the x-axis of the figure. In each case, the optimal response with parameter uncertainty is plotted relative to the optimal response absent uncertainty. Except when $\lambda$ equals or is very close to zero, the degree of attenuation in response to inflation shocks is nearly the same as in the case of output shocks and the degree of precision of the estimate of $\kappa$ is largely irrelevant. If $\lambda$ is equal to or near 0, then the degree of attenuation is greater than in the case of output shocks and the precision of the estimate of $\kappa$ affects the degree of attenuation.

# 3 The Zero Lower Bound and Unconventional Policy

In this section, I extend this model to account for the zero lower bound on nominal interest rates and the presence of a second unconventional policy instrument such as the balance sheet programs used by the Federal Reserve and other central banks in recent years. I start with the introduction of the second unconventional policy instrument. I then turn to the issue of the zero lower bound.

## 3.1 Unconventional Policy

Consistent with theories of how balance sheet policies affect the economy, the second policy instrument is assumed to affect the economy through the same basic transmission channel as the short-term interest rate (see Chung et al. 2011 for a discussion). However, the effect of unconventional policy on the output gap is assumed to be characterized by an additional degree of uncertainty beyond that associated with conventional monetary policy, as discussed in Chung et al. (2011) and Williams (2011). Introducing the second policy instrument yields the following modified equation for the output gap:

$$y = -\eta(r - r^* + \theta b) + u, \tag{14}$$

where $b$ denotes the level of the unconventional policy instrument and the prior distribution of $\theta$ is assumed to be independent of the other model parameters. The level of $b$ is normalized so that $b = 0$ corresponds to the case of the unconventional policy having no effect on the output gap. Note that the unconventional policy's effect on output depends on both $\eta$ and $\theta$. The choice of $b$ is assumed to be linear in the two shocks:

$$b = \Gamma u + \Phi e, \tag{15}$$

where $\Gamma$ and $\Phi$ are coefficients chosen by the policymaker.

Abstracting from the zero lower bound, the two policy instruments are interchangeable if $\theta$ is known with certainty, but are not interchangeable if $\theta$ is uncertain. With the two policy instruments, the expected loss is given by:

$$\mathcal{L} = E\left\{(\kappa^2 + \lambda)(1 - \eta\gamma - \eta\theta\Gamma)^2\sigma_u^2 + [(1 - \kappa(\eta\phi + \eta\theta\Phi))^2 + \lambda(\eta\phi + \eta\theta\Phi)^2]\sigma_e^2\right\}, \tag{16}$$

This yields a set of two optimality conditions for $\gamma$ and $\Gamma$:

$$\gamma^* = \gamma^B - \bar{\theta}\Gamma^*, \tag{17}$$

$$\Gamma^* = \frac{1}{\bar{\theta}(1 + t_\theta^{-2})}(\gamma^B - \gamma^*). \tag{18}$$

The first equation indicates that the optimal value of $\gamma$ equals the Bayesian optimal value adjusted for the effects of the second policy instrument $b$. The second equation indicates that the optimal responsiveness of the second instrument $b$ offsets, up to a dampening factor reflecting uncertainty regarding $\theta$, any deviation of the responsiveness of the first policy instrument $r$ from the optimal level.

In the case of no uncertainty about $\theta$, the two instruments are interchangeable, and there exists a linear combination of values of the two parameters that satisfies these two equations. If $\sigma_\theta > 0$, then the unique optimal policy is given by $\Gamma^* = 0$ and $\gamma^* = \gamma^B$. The logic for this result is straightforward. The instrument $b$ is associated with greater policy error variance than is $r$, so conventional policy strictly dominates unconventional policy.[2]

The corresponding results for the response to the inflation shock are given by:

$$\phi^* = \phi^B - \bar{\theta}\Phi^*, \tag{19}$$

$$\Phi^* = \frac{1}{\bar{\theta}(1 + t_\theta^{-2})}(\phi^B - \phi^*). \tag{20}$$

If $\sigma_\theta > 0$, then the unique optimal policy is given by $\Phi^* = 0$ and $\phi^* = \phi^B$.

### 3.2 The Zero Lower Bound

I now consider the existence of a zero lower bound on $r$.[3] I assume the second instrument $b$ is not constrained in any way. As shown earlier, absent uncertainty about $\theta$, the two instruments are interchangeable, so a constraint on one has no effect on outcomes. In the case of $\sigma_\theta > 0$, the presence of the zero lower bound on $r$ does affect outcomes under optimal policy when the constraint is binding.

If the constraint on $r$ is binding under the optimal policy with parameter uncertainty, then it is optimal to lower $r$ to the limit of the constraint, that is, to zero. The second instrument $b$ is used to make up for the missing stimulus due to the zero lower bound constraint on $r$; however,

[2] In mathematical terms, the variance of the effect of unconventional policy is given by $\bar{\theta}^2\sigma_\eta^2 + \bar{\eta}^2\sigma_\theta^2 + \sigma_\eta^2\sigma_\theta^2$. This is strictly greater than the variance associated with a conventional policy action scaled to have the same mean effect on output, given by $\bar{\theta}^2\sigma_\eta^2$, if $\sigma_\theta^2 > 0$.

[3] It is straightforward to substitute an arbitrary lower bound for the zero lower bound in this analysis.

the response is attenuated owing to the uncertainty regarding $\theta$. Let $r^B$ denote the optimal setting of policy under uncertainty absent the zero lower bound and assuming $b = 0$:

$$r^B \equiv r^* + \gamma^B u + \phi^B e.$$

Then, the optimal policy can be implemented as follows:

$$r = \max\{r^B, 0\}, \tag{21}$$

$$b = \frac{r^B - r}{\bar{\theta}(1 + t_\theta^{-2})}. \tag{22}$$

Note that in this model the presence of the zero lower bound does not create an incentive to deviate from the optimal setting of conventional policy when the constraint is not binding. In other models, there is an incentive to respond to the possibility that the zero lower bound will constrain policy in the future (Reifschneider and Williams 2000).

## 4 Quantifying the Effects of Parameter Uncertainty

The preceding discussion shows that t-statistics associated with key macroeconomic model parameters determine the degree of optimal policy attenuation owing to parameter uncertainty. In this section, I draw on empirical estimates of the relevant parameters and compute the optimal degree of policy attenuation in the model presented in Sections 2 and 3 based on these estimates.

To gauge the effects of parameter uncertainty on optimal conventional monetary policy, I estimate a version of the simple model described earlier. The conventional policy instrument is the federal funds rate. The model is estimated on annual U.S. data from 1960 to 2012. Given the limited sample of unconventional policy use and the reduced-form nature of the stylized model, I am not able to directly estimate the effects of unconventional policy on the economy. Instead, I use estimates of the effects of unconventional policy based on Li and Wei (2013) and Chen et al. (2012), as described later in this section.

To capture the dynamics evident in the data, for the purposes of estimation I modify the stylized model described earlier in four ways.[4] First, I include constants in the two equations that pin down the steady-state inflation rate and nominal interest rate, which are set at 2 and 4 percent,

[4] The empirical version of the model is quite similar to that studied by Svensson (1997).

respectively. Second, I specify a first-order autoregressive structure to the shocks. Third, the right-hand-side variables in each equation are lagged by one year to capture the monetary transmission lag evident in the data. Fourth, I rewrite the inflation equation in terms of the difference between the inflation rate and its time-varying trend.

Specifically, the empirical model takes the following form:

$$y_t = -\eta(r_{t-1} - 4) - \eta\theta b_{t-1} + u_t, \qquad u_t = \rho_u u_{t-1} + \epsilon_{u,t}, \tag{23}$$

$$\pi_t - \pi^*_{t-1} = \text{constant} + \kappa y_{t-1} + e_t, \qquad e_t = \rho_e e_{t-1} + \epsilon_{e,t}. \tag{24}$$

The two shock processes are assumed to be independent of each other. The output gap is measured by the percent deviation of real GDP from the Congressional Budget Office estimate of potential output. The inflation rate is the percent change in the core personal consumption expenditures price index (that excludes food and energy prices). Trend inflation is constructed using the asymmetric band-pass filter.[5] The constant in the inflation equation is set so that the equilibrium inflation rate at the end of the sample is 2 percent.[6]

As I noted, I do not directly estimate the effect of unconventional policy. Instead, I construct a proxy for these effects based on findings from other research. First, I take the Li and Wei (2013) estimates of the effects of the various asset purchase programs on the term premium of the 10-year Treasury note.[7] I then use the Chen et al. (2012) estimate of the effect of a movement in this term premium on the level of real GDP after one year. They find that a 100 basis point decrease in the term premium boosts the level of real GDP by 1.2 percent. This estimate, combined with the estimate of $\eta$, implies a value for $\theta$. Note that the derived value of $\theta$ only affects the size of the model residuals over 2010–2012.

The first line of Table 1 reports the point estimates of the model parameters. The two equations are estimated by least squares with Newey-West heteroskedasticity consistent estimates of the standard errors. The resulting t-statistics are reported in the final line of the table.

[5] The frequencies are 2 and 16 and the data are assumed to be stationary in first differences. The sample used for the band pass filter is 1955–2012.

[6] The unconstrained estimate of the constant implies a steady-state inflation rate just a shade below 2 percent.

[7] Specifically, I assume that Fed asset purchases reduce the 10-year Treasury term premium by 60 basis points in 2009 and 2010, 80 basis points in 2011, and 100 basis points in 2012. These estimates are comparable to those in Chung et al. (2012).

Table 1. Model Estimates

| | $\eta$ | $\kappa$ | $\rho_u$ | $\rho_e$ |
|---|---|---|---|---|
| Estimate | 0.421 | 0.145 | 0.898 | 0.459 |
| standard error | 0.188 | 0.040 | 0.066 | 0.125 |
| t-statistic | 2.237 | 3.648 | | |

The key result for what follows is that the t-statistic associated with $\eta$ is around 2. This value is smaller than the typical, published estimates of comparable t-statistics, which tend to be around 3 (Rudebusch and Svensson 1999, Laubach and Williams 2003).[8] However, these estimates likely overstate the precision of our knowledge of these parameters. In particular, reported standard errors do not account for pre-estimation sample and specification selection, nor do they account for publication bias in empirical research.[9] Arguably, the parsimony of the stylized model specification and long sample should minimize sample- and specification-based biases to the estimated precision of the parameter estimates.

Parameter uncertainty can explain only about 20 percent attenuation relative to the certainty equivalent optimal conventional policy derived assuming no uncertainty about model parameters. Given the values of the t-statistics associated with the estimates of $\eta$ and $\kappa$, I compute the optimal conventional monetary policy under uncertainty, as described in Section 2. The results are reported in the first column of Table 2. Based on the estimated model, the optimal response of the interest rate to an output shock is about 83 percent as large as in the case of no uncertainty. As long as $\lambda$ is not zero, the degree of attenuation for the response to the inflation shock is about the same as for the output gap shock. As discussed in Section 2, the t-statistic for $\kappa$ has little effect on optimal policy as long as $\lambda$ is nontrivially above 0. In the case of $\lambda$ close to or equal to zero, the degree of attenuation slightly exceeds 20 percent.

[8]Similarly, Levin et al. (2005) finds policy attenuation due to parameter uncertainty of only about 10 percent, which corresponds to the optimal attenuation in the stylized model with t-statistics associated with $\eta$ of 3 described earlier.

[9]In addition, Bayesian estimation techniques have become increasingly common in estimating macroeconomic models. The reported precision of parameter estimates using Bayesian methods are influenced by the choice of the prior distribution, which may not have a strong grounding in theory or evidence. These arguments suggest that published estimates of the relevant t-statistics may overstate the accuracy of parameter estimates.

Table 2. Policy Attenuation from Parameter Uncertainty

| | Conventional Policy | Unconventional Policy |
|---|---|---|
| $\gamma^B/\gamma^C$ | 0.83 | 0.50 |
| $\phi^B/\phi^C$ | | |
| $\lambda = 0$ | 0.78 | 0.47 |
| $\lambda = 0.25$ | 0.83 | 0.50 |
| $\lambda = 1$ | 0.83 | 0.50 |

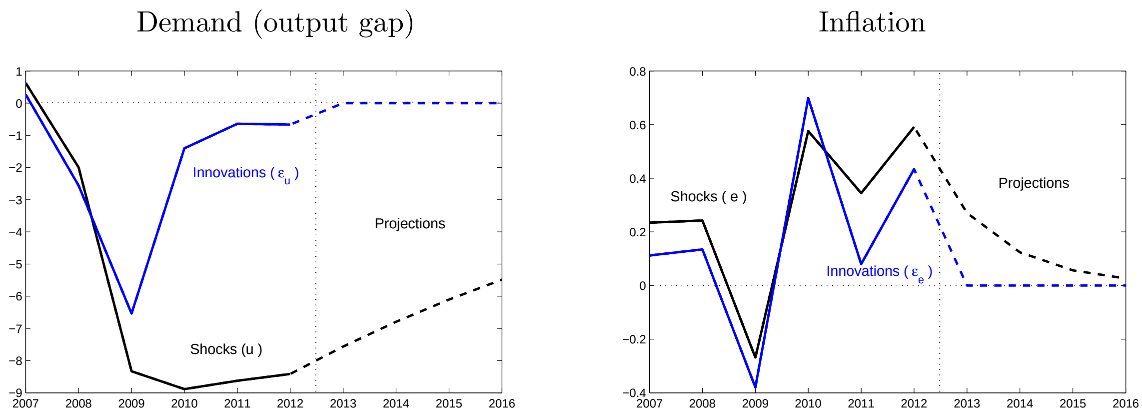
Turning to unconventional monetary policy, estimation of $\theta$ in this simple model is impractical owing to the small number of observations of balance sheet policies and the difficulties in identification. Instead, I rely on estimates gleaned from structural models that distinguish between conventional and unconventional monetary policy actions. In particular, I measure the uncertainty regarding $\theta$ using estimates of the effects of unconventional policy on output from Chen et al. (2012). These authors find that the t-statistic associated with the effect of unconventional policy on output is 1.0, and that for inflation is 1.1.[10] Clearly, the effects of unconventional policy are subject to far greater uncertainty than those of conventional monetary policy (see also Williams 2011, 2012).

Based on these estimates, the optimal degree of attenuation for unconventional policy is very large: 50 percent, reflecting the high degree of uncertainty regarding the effects of unconventional policy. I compute the implied value of $\sigma_\theta$ by combining the estimate of $\eta$ from the empirical model described earlier and the t-statistic of unity estimated by Chen et al. (2012). Assuming the two parameters $\eta$ and $\theta$ are independent of each other, the implied value of $\sigma_\theta$ is 1.2245.[11] Given this estimate of $\sigma_\theta$, I compute the optimal policy responses for the unconventional policy instrument according to the model described in Section 3. The second column of Table 2 reports the degree of attenuation implied by the model estimates.

[10] See also Kiley (2012) and Baumeister and Benati (2010) for estimates of heightened uncertainty regarding the effects of unconventional monetary policy.

[11] If I instead use the estimated t-statistic for the effect of unconventional policy on inflation, the implied value of $\sigma_\theta$ is approximately 1.55. Using this value would have only a modest effect on the results reported in this paper.

**Figure 2: Model Shocks**




Notes: The shocks are derived from the estimated model as described in the text.

# 5 Optimal Policy in the Current Economic Environment

In this section, I use simulations of the empirical model described in the previous section to illustrate the effects of parameter uncertainty on the setting of optimal monetary policy in the current economic environment. Of course, these results are specific to the simple stylized model I am using. As such, the quantitative results regarding the projected paths should be viewed as only illustrative. Despite these limitations, this analysis highlights some key issues in thinking about optimal policy in the current circumstances.

Figure 2 shows the demand and inflation shocks and the underlying innovations implied by the estimated model from 2007 to 2012. The demand shock innovations are negative and large in 2008–2010, and modestly negative in 2011 and 2012. As a consequence, the demand shock is massively negative in 2009 (a nearly 4 standard deviation innovation according to the model) and becomes only modestly less negative through 2012. During the projection period starting in 2013, the shock innovations are assumed to be zero and the demand shock gradually dissipates. The inflation shocks are positive in each year shown in the chart except for 2009. This pattern of positive inflation shocks reflects the relative stability of core inflation in the vicinity of 2 percent over this period despite the presence of large negative output gaps. At the start of the projection period, the inflation shock is positive and sizable, but dissipates quickly owing to the relatively small estimated autoregressive coefficient of this process.

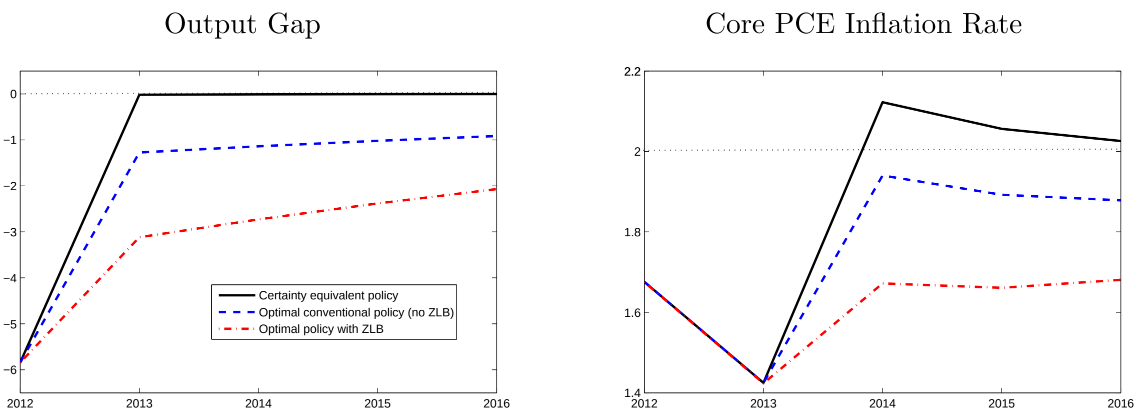
I examine three alternative monetary policies derived in the preceding section. The first is the certainty equivalent policy, where the two policy instruments are used assuming there is no uncertainty about $\eta$, $\kappa$, and $\theta$. The second policy is the optimal policy under uncertainty based on the degree of uncertainty associated with conventional policy alone. That is, this policy represents the optimal policy if there were no zero lower bound (or, equivalently, if the policymaker assumes $\sigma_\theta = 0$). The third policy is the optimal policy under uncertainty with the zero lower bound imposed on the short-term interest rate. This takes into account the estimated degree of uncertainty about $\eta$, $\kappa$, and $\theta$.

In implementing the alternative policies, I need to adjust the specification of the policies to account for the lagged structure of the empirical model. Because policy affects output with a one-period lag, optimal policy responds to the expected level of the demand shock in the next period rather than the current level of the shock. This is equivalent to multiplying the policy response parameter $\gamma$ by the autocorrelation parameter of the demand shock, $\rho_u$. Similarly, policy affects the inflation rate with a two-period lag, so policy responds to the expected value of the inflation shock two periods ahead. This is equivalent to multiplying the policy response parameter $\phi$ by the square of the autocorrelation parameter of the inflation shock, $\rho_e$. Note that uncertainty about the autocorrelations of the shocks does not affect the optimal response to shocks. In the remainder of the paper I assume that these model parameters are fixed and known.

I assume that the policymaker does not learn about the effectiveness of policy, but rather has fixed priors over the point estimates and uncertainty regarding model parameters. I start each simulation in 2012 with the setting of monetary policy in that year. The first period of simulated output and inflation data is 2013; the final period is 2016. The simulations are conducted using the point estimates of the model parameters and assuming the innovations are zero in the simulation periods. Figure 3 shows the simulated output gap and inflation data for the three alternative policy regimes assuming equal weights on the output gap and inflation in the central bank loss, that is, $\lambda = 1$.

The certainty equivalent policy aims to drive the output gap very close to zero in 2013 and keep it there in ensuing years. Given the history of positive inflation shocks, this policy allows

**Figure 3: Model Projections under Alternative Policies ($\lambda = 1$)**



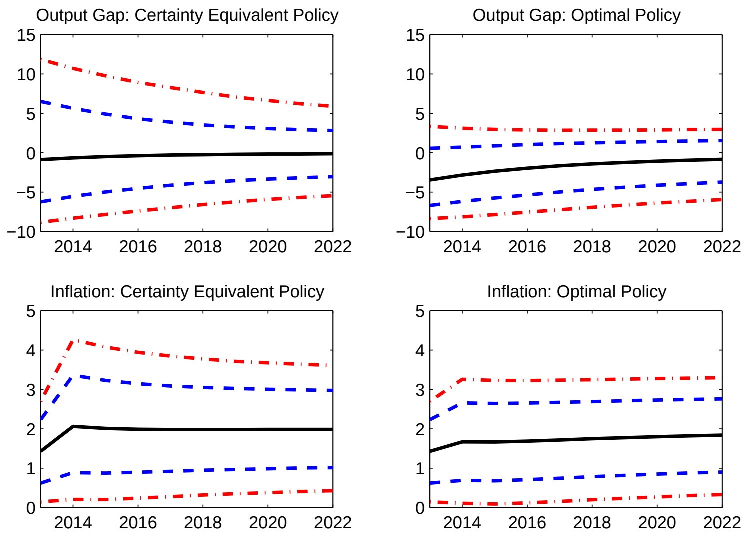

Notes: In the simulations, the demand and inflation shock innovations are assumed to be zero in 2013 and thereafter. The model parameters are set to their respective prior mean values.

inflation to temporarily rise above the 2 percent long-run target and creates a very small negative output gap to modestly offset inflationary pressures. The pattern of outcomes under the certainty equivalent policy is consistent with the claim that more aggressive policy could close the output gap quickly and bring inflation back to target (or even overshoot the target for a while). Indeed, assuming the point estimates of the model parameters are correct, this policy would yield superior outcomes in this model.

The certainty equivalent policy is *not* optimal once one recognizes parameter uncertainty. The blue dashed lines in Figure 3 show the outcomes under the optimal policy that accounts for parameter uncertainty associated with conventional policy. That is, this policy is based on the assumption that $\sigma_\theta = 0$. The moderate degree of policy attenuation causes the simulated paths for both the output gap and the inflation rate to remain somewhat below long-run target levels. Accounting for the greater degree of uncertainty regarding unconventional policy actions, the simulated paths of the output gap and inflation, shown by the dash-dot line in the figure, are persistently well below target levels for years.

The finding that the optimal policy under uncertainty is expected to persistently undershoot both output and inflation objectives may seem counterintuitive at first. The key insight is that the projected path assuming known parameters does not adequately summarize the policy choices and

**Figure 4: Distributions of Outcomes Under Alternative Policies ($\lambda = 1$)**

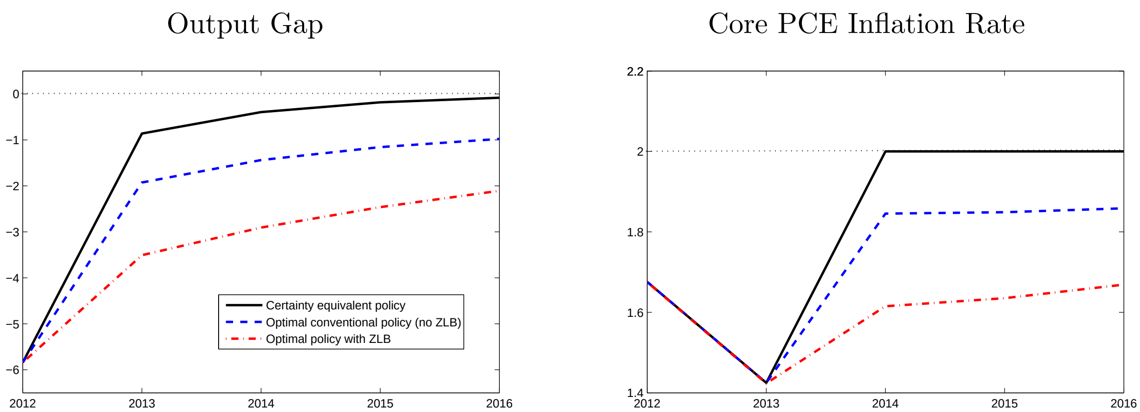



Notes: The black solid lines show the median responses. The blue dashed lines show the boundaries of the 70 percent confidence bands. The red dot-dash lines show the boundaries of the 90 percent confidence bands.

potential outcomes that the policymaker faces because it ignores the effects of parameter uncertainty. To provide a more complete picture of the outcomes under different policies, Figure 4 shows the distributions of outcomes under the certainty equivalent policy and the optimal policy under uncertainty for 2013–2022. For these calculations, I repeatedly simulate the model as described earlier, drawing random demand and inflation shocks and values of the model parameters each period. For this purpose, I assume all random variables are distributed normally with means and variances given by the priors described earlier. I simulate the model one million times.

The certainty equivalent policy generates highly variable outcomes owing to the aggressive use of unconventional policy, especially during the early years of the simulations. The left-hand panels of the figure show the distributions of outcomes under the certainty equivalent policy. Although the median responses, shown by the black solid lines, are close to targeted levels, the range of outcomes is huge, reflecting the estimated high degree of uncertainty regarding the effects of unconventional policy. The outcomes under the optimal policy are shown in the right-hand panels of the figure. Although the median responses under the optimal policy undershoot the target levels,

**Figure 5: Model Projections under Alternative Policies ($\lambda = 0$)**




Notes: In the simulations, the demand and inflation shock innovations are assumed to be zero in 2013 and thereafter. The model parameters are set to their respective prior mean values.

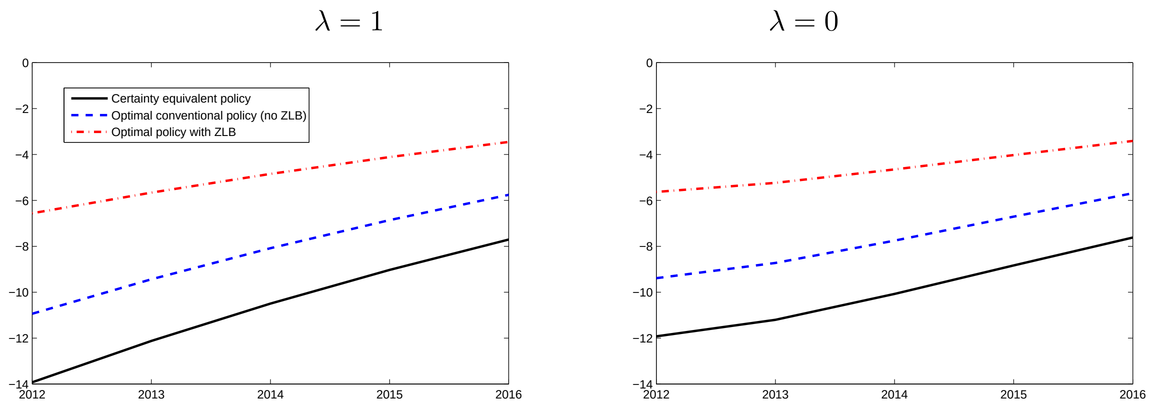
the distribution of outcomes is far tighter than under the certainty equivalent policy.

The expected loss under the certainty equivalent policy is significantly larger than under the optimal policy. Based on the model simulations, the expected loss under the certainty equivalent policy, computed over 2013–2022, is 25.4. The corresponding figure for the optimal policy under uncertainty is 14.8. For comparison, the loss under the policy that assumes $\sigma_\theta = 0$ is 18.4.

These results are not sensitive to the weight on output stabilization in the central bank objective function. Figure 5 reports the model projections for the output gap and the inflation rate for the three representative policies assuming $\lambda = 0$. The certainty equivalent policy drives the inflation rate to the target level of 2 percent as soon as possible and keeps it there (in expectations, assuming the model parameters equal their prior mean values). This necessitates creating a moderately negative output gap each year that decreases in magnitude with each passing year.

Although parameter uncertainty calls for a significant degree of attenuation in the policy response relative to the certainty equivalent policy, the optimal policy under uncertainty nonetheless calls for prolonged sizable monetary stimulus. Figure 6 shows the stance of monetary policy, defined to be $r + \bar{\theta}b$, for the three policies and two values of $\lambda$. In all cases, the short-term rate of interest is zero, and unconventional policies are in use through 2016 (the end of the simulation). Interestingly, the optimal policy accounting for parameter uncertainty and the zero bound is relatively insensitive

**Figure 6: Monetary Policy Stance ($r + \bar{\theta}b$)**




Notes: In the simulations, the demand and inflation shock innovations are assumed to be zero in 2013 and thereafter. The model parameters are set to their respective prior mean values.

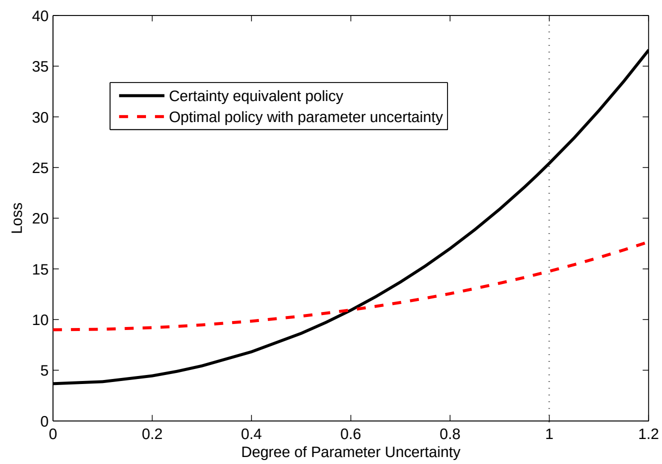
to the weight on output stabilization in the objective function, $\lambda$. I should emphasize that these quantitative results on the optimal stance of policy in the current situation are sensitive to the specification of the stylized model and, in particular, to the estimate of the magnitude of the demand shock (which itself depends on the estimated size of the output gap).

## 5.1 Sensitivity Analysis

Up to this point, the analysis has taken the model specification and prior distributions of parameters as given. I now consider the sensitivity of the results to deviations from the baseline model. I first consider sensitivity to the degree of parameter uncertainty and then consider modifications to the model specification.

Figure 7 shows the expected losses under the two alternative policies for differing degrees of parameter uncertainty based on stochastic simulations of the model as described above and assuming $\lambda = 1$. The loss is computed over 2013–2022. The solid black line shows the simulated expected losses under the certainty equivalent policy for different degrees of parameter uncertainty. The dashed line shows the results under the optimal policy based on the baseline estimates of parameter uncertainty. The values on the x-axis indicate the amount by which the standard deviations of the model parameters are multiplied, relative to the baseline values. That is, a value of unity represents the baseline prior distributions for the parameters.

**Figure 7: Sensitivity to Degree of Parameter Uncertainty ($\lambda = 1$)**




Notes: The solid black line shows the simulated expected losses under the certainty equivalent policy for different degrees of parameter uncertainty. The dashed line shows the results under the optimal policy based on the baseline estimates of parameter uncertainty. The values on the x-axis indicate the amount by which the standard deviations of the model parameters are multiplied.

The optimal policy based on the baseline degree of parameter uncertainty provides insurance against uncertainty about the effects of policy on output and inflation. As seen in Figure 7, the expected loss under the certainty equivalent policy is lower than that under the baseline optimal policy if the degree of parameter uncertainty is less than 60 percent of the baseline estimates. Put differently, the optimal policy under the baseline parameter uncertainty provides insurance against moderate degrees of parameter uncertainty, purchased at the cost of somewhat worse performance if parameter uncertainty is very low.

I now examine the sensitivity of the results to modifications in the model specification. Craine (1979) and Söderström (2002) show that the standard Brainard policy attenuation result depends on the model specification and sources of uncertainty. A full analysis of this issue is beyond the scope of this paper. Instead, I consider two modifications to the model specification and examine the effects of uncertainty on monetary policy in each case. In particular, I allow for intrinsic inertia in the inflation and output gap processes. These modifications imply changes in the specification of the certainty equivalent and optimal policy under uncertainty. However, to facilitate comparison with the results obtained under the baseline model, I restrict the analysis to the same linear policy reaction function as before. I numerically compute the coefficients of this reaction function that minimize the central bank expected loss. That is, I examine optimized reaction functions, rather than the fully optimal policies in the modified versions of the model.

I first introduce intrinsic inertia in the inflation process. I start with the lagged structure of the empirical version of the stylized model described in section 4. In particular, I specify a modified inflation equation of the following form:

$$\pi_t - \pi^*_{t-1} = \delta_\pi(\pi_{t-1} - \pi^*_{t-1}) + \kappa y_{t-1} + e_t, \qquad e_t = \rho_e e_{t-1} + \epsilon_{e,t}. \tag{25}$$

I consider various degrees of intrinsic inflation inertia, as measured by $\delta_\pi$. All other parameter values and associated prior distributions are assumed to be the same as in the baseline model. Given this modified equation and the other model equations, I numerically compute the optimal values of $\phi$ and $\gamma$, for both conventional and unconventional policies, based on the estimated unconditional central bank loss from stochastic simulations (each of one million periods) of the model. The exercise is conducted both for the certainty equivalent policy and the optimal policy under uncertainty. I assume that the coefficient on lagged inflation is fixed and known. I further assume that the inflation target is constant at 2 percent. The results are shown in Table 3. The corresponding results for the baseline model are shown for comparison in the upper part of the table.

The addition of intrinsic inflation inertia does not materially change the results regarding the attenuation effects of parameter uncertainty on the optimal policy. The optimal policy of the modified model responds to the output gap shock nearly the same as for the baseline model. The optimal response to the inflation shock is larger than in the baseline model, but the degree of attenuation associated with uncertainty is about the same.

Table 3. Sensitivity to Model Specification

| | Conventional Policy | | Unconventional Policy | |
|---|---|---|---|---|
| | $\gamma$ | $\phi$ | $\gamma$ | $\phi$ |
| Baseline model ($\delta_\pi = \delta_y = 0$) | | | | |
| Certainty equivalent policy | 2.14 | 0.07 | 2.14 | 0.07 |
| Optimal policy | 1.78 | 0.06 | 1.07 | 0.04 |
| Inflation inertia ($\delta_\pi = 0.25, \delta_y = 0$) | | | | |
| Certainty equivalent policy | 2.14 | 0.15 | 2.14 | 0.15 |
| Optimal policy | 1.79 | 0.13 | 1.08 | 0.08 |
| Inflation inertia ($\delta_\pi = 0.50, \delta_y = 0$) | | | | |
| Certainty equivalent policy | 2.14 | 0.37 | 2.14 | 0.37 |
| Optimal policy | 1.81 | 0.30 | 1.10 | 0.19 |
| Output gap inertia ($\delta_y = 0.25, \delta_\pi = 0$) | | | | |
| Certainty equivalent policy | 2.23 | 0.06 | 2.23 | 0.06 |
| Optimal policy | 1.99 | 0.06 | 1.36 | 0.04 |
| Output gap inertia ($\delta_y = 0.50, \delta_\pi = 0$) | | | | |
| Certainty equivalent policy | 2.29 | 0.05 | 2.29 | 0.05 |
| Optimal policy | 1.99 | 0.06 | 1.66 | 0.03 |

I now consider a similar modification to the output gap equation. In particular, I assume the following:

$$y_t = \delta_y y_{t-1} - \eta(r_{t-1} - 4) - \eta\theta b_{t-1} + u_t, \qquad u_t = \rho_u u_{t-1} + \epsilon_{u,t}. \tag{26}$$

I include this modified output gap equation in the baseline model (including the baseline inflation equation). The results are shown in the lower part of Table 3.

Incorporating output gap intrinsic inertia increases the optimal response to demand shocks and tempers somewhat the degree of attenuation caused by parameter uncertainty. For example, in the baseline model, uncertainty causes a 50 percent reduction in the response of unconventional policy to a demand shock. In the modified model, the degree of attenuation is about 40 percent. The optimal response to an inflation shock is about the same as in the baseline model. If I allow for even greater output gap inertia, the degree of policy attenuation declines further. For example, with a coefficient on the lagged output gap of 0.5, the optimal unconventional policy response to a demand shock is 28 percent less than for the certainty equivalent policy.[12]

This analysis confirms the finding that the degree of optimal policy attenuation due to uncertainty can be sensitive to model specification. However, given the large estimated degree of uncertainty around unconventional policies, it is still the case that uncertainty calls for significant attenuation in the use of these policies.

## 6 Unmodeled Costs of Unconventional Policies

Up to this point, I have assumed there are no constraints or costs associated with the use of unconventional policy. However, some commentators believe these policies may have unintended consequences, such as contributing to the risk of financial instability or unmooring inflation expectations, that are not captured by the model I have used.

Little research has been done to explore the potential costs of unconventional policies. Therefore, I do not model these costs directly but instead take a short-cut and assume that the policymaker associates increasing costs with the use of unconventional policies.[13] Specifically, I posit the modified policymaker objective function:

$$\mathcal{L} = E\{(\pi - \pi^*)^2 + \lambda y^2 + \psi b^2\}, \tag{27}$$

where $\psi$ indicates the magnitude of the costs of using unconventional policy.

The presence of quadratic costs with the size of unconventional policy affects the optimal setting of unconventional policy in the same way as uncertainty regarding the effectiveness of this policy. To make this relationship clear, I abstract from parameter uncertainty to derive the optimal policy that includes costs from the use of unconventional policy. The resulting optimality condition for conventional and unconventional policy is given by:

$$r = \max\{r^C, 0\}, \tag{28}$$

$$b = \left(\frac{r^C - r}{\bar{\theta}}\right)\left(\frac{1}{1 + \frac{\psi}{(\lambda+\bar{\kappa}^2)\bar{\eta}^2}}\right), \tag{29}$$

[12] Although not reported in the table, allowing for uncertainty regarding $\delta_\pi$ and $\delta_y$ does not affect the optimal value of $\gamma$. Uncertainty about $\delta_\pi$ somewhat increases the optimal value of $\phi$.

[13] Such a penalty on policy actions has often been used in analysis of conventional monetary policy. See Williams (2003) and Woodford (2003) for examples and references.

where $r^C$ is the optimal setting of the short-term interest rate ignoring uncertainty and the zero lower bound. Note that the optimal policy is to first deploy the short-term interest rate to the fullest extent possible, and then use unconventional policy according to this equation if needed. Thus, a concern for costs of unconventional policy not captured by the model mimics (or reinforces) policy attenuation due to parameter uncertainty.

An additional concern about unconventional policies is that *changes* in these policies may be disruptive and engender unmodeled costs. Analysis of such costs is beyond the scope of this paper. However, one implication of this concern for deploying unconventional policy is clear. It argues for gradualism in shifts in the stance of policy, both as the policies are increased and are reduced. Indeed, such a concern would run counter to the strong result from the stylized model that unconventional policies should only be deployed when conventional policy is fully utilized.

# 7 Conclusion

The analysis presented here shows how optimal monetary policy that accounts for parameter uncertainty may appear to be suboptimal when uncertainty is ignored. The paper's basic approach to parameter uncertainty is not novel and is found in the theoretical analysis of Brainard (1967), the empirical studies of Federal Reserve monetary policy by Rudebusch (2001) and Sack (2000), and Blinder's (1998) account of monetary policy in practice. In the current circumstances, where the Federal Reserve and other central banks rely on unconventional policy tools, uncertainty regarding the effects of policy actions is particularly relevant. Although there is compelling evidence that the Fed's balance sheet policies have reduced long-term interest rates, considerable uncertainty remains regarding their effects on broader financial conditions, economic activity, and inflation. It is difficult to quantify the uncertainty regarding the macroeconomic effects of balance sheet policies. However, evidence suggests that the uncertainty regarding their effects is considerably larger than that of conventional monetary policy.

This analysis is subject to a number of caveats that point to directions for further research. First, as discussed in Craine (1979) and Bernanke (2004), the effects of uncertainty on monetary policy depend on the model and the sources of uncertainty. The model used here is highly stylized and abstracts from expectations channels. A more thorough analysis using a richer dynamic macroeconomic model is needed to better quantify the effects of uncertainty. Such an analysis should also consider a broader set of sources of uncertainty, including model misspecification (see, for example, Orphanides and Williams, 2006, and references therein). Second, the analysis presented here ignores the ability of the policymaker to learn over time. In an environment where the policymaker can learn, there is an incentive to experiment, which works in opposition to the attenuation motive analyzed here (see Sack 1998, and Wieland, 2000, and references therein). Third, the analysis takes the policymaker objective function as exogenously given. Levin and Williams (2003) show that the attenuation effects of parameter uncertainty are reduced in some cases when the central bank's loss is explicitly defined to be the welfare of the representative agent (see also Levin et al. 2006, and Edge et al. 2010).

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