Complex Systems Workshop
Lecture III: Behavioral Asset Pricing Model with Heterogeneous Beliefs

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Outline

1 Introduction

2 The model
   - 2-type model: fundamentalists vs. trend followers
   - 3-type model: fundamentalists vs. optimists and pessimists
   - 4-type model: fundamentalists vs. trend and bias

3 Empirical Validation


Traditional Rational View

- representative agent, who is perfectly rational
- expectations are model consistent
- Friedman hypothesis: “irrational agents will lose money and will be driven out the market by rational agents”
- simple (linear), stable model, driven by exogenous random news about fundamentals
- prices reflect economic fundamentals (market efficiency)
- Lucas: macroeconomic policy should be based on rational expectations
Heterogeneous, interacting agents approach

- **heterogeneous** agents, heterogeneous beliefs
- market **psychology, herding** behavior (Keynes (1936))
- **bounded rationality** (Simon (1957))
- markets as **complex adaptive, nonlinear evolutionary** systems
- interactions of agents create **aggregate structure** explaining stylized facts
Some Problems Interacting Agents Approach

- ‘wilderness’ of bounded rationality
- many degrees of freedom for heterogeneity
- what exactly causes the outcome in a (large) computational HAM
How to Discipline Bounded Rationality?

- **Stylized** agent-based models
- **Behavioral rationality** – behavioral consistency: simple heuristics that work reasonably well
- **Evolutionary selection** (‘survival of the fittest’) and reinforcement learning
- **Laboratory experiments** to test individual decision rules and aggregate macro behavior
Asset Pricing Model with Homogeneous Beliefs

Agents choose between risk free and risky asset:

- \( R = 1 + r > 1 \): gross return on risk free asset
- \( p_t \): price (ex div.) per share of risky asset
- \( y_t \): IID dividend process for risky asset
- \( z_t \): number of shares purchased at date \( t \)

End of period wealth:

\[
W_{t+1} = R(W_t - p_t z_t) + (p_{t+1} + y_{t+1})z_t = RW_t + (p_{t+1} + y_{t+1} - Rp_t)z_t
\]

Myopic mean variance maximization: demand \( z_t \) solves

\[
\text{Max}\left\{ E_t W_{t+1} - \frac{a}{2} V_t W_{t+1} \right\}, \quad \text{so}
\]

\[
z_t = \frac{E_t[p_{t+1} + y_{t+1} - Rp_t]}{aV_t[p_{t+1} + y_{t+1} - Rp_t]} = \frac{E_t[p_{t+1} + y_{t+1} - Rp_t]}{a\sigma^2}
\]

(where \( \sigma^2 \) is common beliefs on variance \( V_{ht} \)).
Equilibrium Price

Market equilibrium between supply and demand:

\[
E_t(p_{t+1} + y_{t+1} - R\rho_t) \frac{1}{a\sigma^2} = z^s
\]

equilibrium pricing equation:

\[
R\rho_t = E_{ht}(p_{t+1} + y_{t+1}) - a\sigma^2 z^s
\]

special case: constant zero supply of outside shares \( z^s = 0 \):

\[
R\rho_t = E_{ht}(p_{t+1} + y_{t+1})
\]
REE fundamental solution

Equilibrium pricing equation:
(common beliefs on future dividends $E_t[y_{t+1}]$)

$$ Rp_t = E_t[p_{t+1} + y_{t+1}] $$

“no bubble” condition implies unique bounded fundamental solution $p_t^*$:
(discounted sum expected future cash flow)

$$ p_t^* = \frac{E_t[y_{t+1}]}{R} + \frac{E_t[y_{t+2}]}{R^2} + \ldots $$

For special case of IID dividends, with $E_t[y_{t+1}] = \bar{y}$:

$$ p_t^* = \frac{\bar{y}}{R - 1} = \frac{\bar{y}}{r} $$
Model in deviations from fundamental

deviation from fundamental

\[ x_t = p_t - p^* \]

Pricing equation in deviations:

\[ R x_t = E_t x_{t+1} \]

Notice: rational bubble solutions: \( x_t = x_0 R^t \)

with self-fulfilling belief \( x_{t+1}^e = g x_{t-1} \), with \( g = R^2 \).

Equilibrium pricing equation with heterogeneous beliefs:
(in deviations from RE-fundamental)

\[ R x_t = \sum_{h=1}^{H} n_{ht} E_{ht} x_{t+1} \]
Asset pricing model with heterogeneous beliefs

agents choose to invest in risk free or risky asset
- $R = 1 + r > 1$: gross return on risk free asset
- $p_t$: price (ex div.) per share of risky asset
- $y_t$: IID dividend process for risky asset
- $n_{ht}$: fraction of agents of type $h$

Myopic mean variance maximization of expected wealth demand for risky asset by type $h$:

$$z_{ht} = \frac{E_{ht}[p_{t+1} + y_{t+1} - Rp_t]}{aV_{ht}[p_{t+1} + y_{t+1} - Rp_t]} = \frac{E_{ht}[p_{t+1} + y_{t+1} - Rp_t]}{a\sigma^2}$$

(common beliefs on variance $V_{ht} = \sigma^2$ and a risk aversion parameter)
Market equilibrium

Equilibrium of supply and demand:

\[
\sum_{h=1}^{H} n_{ht} \frac{E_{ht}[p_{t+1} + y_{t+1} - R_{p_t}]}{a\sigma^2} = z^s
\]

equilibrium pricing equation:

\[
R_{p_t} = \sum_{h=1}^{H} n_{ht} E_{ht}(p_{t+1} + y_{t+1}) - a\sigma^2 z^s
\]

special case: constant zero supply of outside shares \(z^s = 0\):

\[
R_{p_t} = \sum_{h=1}^{H} n_{ht} E_{ht}(p_{t+1} + y_{t+1}) + \epsilon_t
\]

(where noise term \(\epsilon_t\) (e.g. random supply of shares) has been added)
Heterogeneous Beliefs

Assumptions about beliefs of trader type $h$:

**B1** same constant beliefs on variances for all types $h$:

$$V_{ht}[p_{t+1} + y_{t+1} - Rp_t] = V_t[p_{t+1} + y_{t+1} - Rp_t] = \sigma^2$$

**B2** common and correct beliefs on future dividends:

$$E_{ht}[y_{t+1}] = E_t[y_{t+1}], \text{ for all types } h$$

special case of IID dividends:

$$E_{ht}[y_{t+1}] = E_t[y_{t+1}] = \bar{y}.$$  

**B3** heterogeneous beliefs on future prices of the form:

$$E_{ht}[p_{t+1}] = E_t[p^*_t + 1] + E_{ht}[x_{t+1}] = p^*_{t+1} + f_h(x_{t-1}, \cdots x_{t-L})$$

special case of IID dividends:

$$E_{ht}[p_{t+1}] = p^* + f_h(x_{t-1}, \cdots x_{t-L})$$

Under assumptions B1-B3 **equilibrium pricing equation in deviations** $x_t = p_t - p^*$ from the fundamental:

$$Rx_t = \sum_{h=1}^{H} n_{ht} E_{ht}[x_{t+1}] = \sum_{h=1}^{H} n_{ht} f_{ht}$$
Forecasting rules

belief of type $h$ on future prices:

$$E_{ht}[p_{t+1}] = p^* + f_h(x_{t-1}, \ldots, x_{t-L})$$

or in deviations:

$$E_{ht}[x_{t+1}] = f_h(x_{t-1}, \ldots, x_{t-L})$$

Important special cases:

- **rational expectations**: "$f(x_{t-1}, \ldots, x_{t-L}) = x_{t+1}$"

  (also **perfect foresight** on other belief-fractions $n_{ht}$)

- **fundamentalists**: $f \equiv 0$

  (no knowledge about other beliefs and fractions $n_{ht}$)

- **pure trend chasers**: $f(x_{t-1}, \ldots, x_{t-L}) = gx_{t-1}$

- **pure bias**: $f(x_{t-1}, \ldots, x_{t-L}) = b$.

- **simple example**: linear forecast with one lag $f_{ht} = g_h x_{t-1} + b_h$

- **trend extrapolator**: $f_{ht} = x_{t-1} + g_h (x_{t-1} - x_{t-2})$

**Question:**

Do rational agents and/or fundamentalists drive out trend chasers and biased beliefs?
Evolutionary selection of strategies

**Evolutionary selection** or **reinforcement learning**: more successful strategies attract more followers.

**Fractions of belief types** are updated in each period, according to (discrete choice model, BH 1997, 1998)

\[
    n_{ht} = \frac{e^{\beta U_{h,t-1}}}{Z_{t-1}}
\]

where \( Z_{t-1} \) is normalization factor and \( \beta \) is **intensity of choice**.

- \( \beta = 0 \): all types equal weight
- \( \beta = \infty \): “neoclassical limit”, i.e. all agents use best predictor
Evolutionary selection of strategies

**realized profits** in period $t$

$$\pi_{ht} = R_t z_{h,t-1} = (p_t + y_t - R p_{t-1}) \frac{E_{h,t-1}[p_t + y_t - R p_{t-1}]}{a \sigma^2}$$

$$(x_t - R x_{t-1} + \delta_t) \frac{E_{h,t-1}[x_t - R x_{t-1}]}{a \sigma^2}$$

(with $y_t = \bar{y} + \delta_t$)

**Fitness function** or **performance measure**

(weighted sum of) realized profits

$$U_{ht} = \pi_{ht} + w U_{h,t-1} - C_h$$

where $C_h \geq 0$ are **costs** for predictor $h$, and $w$ is **memory strength**

($w = 1$: infinite memory; fitness $\equiv$ accumulated wealth

$w = 0$: memory one lag; fitness $\equiv$ most recently realized net profit)
The model

Asset Pricing Model with Heterogeneous Beliefs

(in deviations from the RE fundamental)

\[ Rx_t = \sum_{h=1}^{H} n_{ht} f_h(x_{t-1}, \ldots, x_{t-L}) = \sum_{h=1}^{H} n_{ht} f_{ht} \]

\[ n_{ht} = \frac{e^{\beta U_{h,t-1}}}{\sum_{h=1}^{H} e^{\beta U_{h,t-1}}} \]

\[ U_{h,t-1} = (x_{t-1} - Rx_{t-1}) \frac{f_{h,t-2} - Rx_{t-2}}{\alpha \sigma^2} - C_h \]
Examples with Heterogeneous Linear Beliefs

(in deviations from the RE fundamental)

**Example** with linear predictors $f_{ht} = g_h x_{t-1} + b_h$:

\[
R_{x_t} = \sum_{h=1}^{H} n_{ht} (g_h x_{t-1} + b_h)
\]

\[
n_{ht} = \frac{e^{\beta U_{h,t-1}}}{\sum_{h=1}^{H} e^{\beta U_{h,t-1}}}
\]

\[
U_{h,t-1} = (x_{t-1} - R_{x_{t-2}}) \left( \frac{g_h x_{t-3} + b_h - R_{x_{t-2}}}{a \sigma^2} \right) - C_h
\]
Two-type Example: Fundamentalists versus trend

Two trader types, with forecasting rules

\[ f_{1t} = 0, \quad \text{fundamentalists at costs C} \]
\[ f_{2t} = gx_{t-1}, \quad g > 0, \quad \text{trend followers} \]

Define difference in fractions: \( m_t = n_{1t} - n_{2t} \)

\[ Rx_t = \frac{1-m_t}{2} gx_{t-1} \]

\[ m_{t+1} = \tanh\left(\frac{\beta}{2} \left[-\frac{gx_{t-2}}{a\sigma^2}(x_t - Rx_{t-1}) - C\right]\right) \]
Two-type Example: Fundamentalists versus trend

**Theorem.** (existence and stability steady states)

\[ m^{eq} = \tanh(-\beta C/2) \]

\[ m^* = 1 - 2R/g \]

Let \( x^* \) be positive solution of

\[ \tanh\left(\frac{\beta}{2} \left[ \frac{g}{a\sigma^2} (R - 1)(x^*)^2 - C \right] \right) = m^* \]

1. \( 0 < g < R \): \( E_1 = (0, m^{eq}) \) globally **stable** steady state
2. very strong trend chaser, i.e \( g > 2R \): **three** steady states \( E_1 = (0, m^{eq}), E_2 = (x^*, m^*) \) and \( E_3 = (-x^*, m^*) \)
3. \( R < g < 2R \): if costs \( C > 0 \), then we have a **pitchfork bifurcation** for \( \beta = \beta^* \), that is, a unique steady state for \( \beta < \beta^* \) and three steady states for \( \beta > \beta^* \).
Moreover, **Hopf bifurcation** of non-fundamental steady states $E_2$ and $E_3$:
- stable for $\beta^* < \beta < \beta^{**}$
- unstable for $\beta > \beta^{**}$

**Corollary**: costly fundamentalists and cannot drive out trend chasers driven by short run profits.
Bifurcation diagram and Lyapunov exponent plot

Figure: Bifurcation diagram (left) and largest Lyapunov exponent plot (right) for 2-type model with costly fundamentalist versus trend followers.
Time series of prices and fractions and attractors

Figure: Time series of prices and fractions and attractors in the phase space for 2-type model with costly fundamentalist versus trend followers.
Homoclinic Orbit for $\beta = +\infty$

Two possibilities for the unstable manifold $W^u(E)$ (Brock and Hommes, 1998, Lemma 4, p.1251):

1. if $g > (1 + r)^2$, then unstable manifold $W^u(E)$ equals the (unbounded) unstable eigenvector; typical solutions are exploding, diverging to infinity with trend followers dominating the market;

2. if $1 + r < g < (1 + r)^2$, then unstable manifold $W^u(E)$ is bounded; all time paths converge to the (locally) unstable saddle-point fundamental steady state; homoclinic orbits
Three Type Example

Three types (zero costs)

\[ f_{1t} = 0 \quad \text{fundamentalists} \]
\[ f_{2t} = b \quad b > 0, \quad \text{positive bias (optimists)} \]
\[ f_{3t} = -b \quad -b < 0, \quad \text{negative bias (pessimists)} \]

\[ R_{xt} = n_{2,t}b_2 + n_{3,t}b_3 \]

\[ n_{j,t+1} = \exp\left(\frac{\beta}{a\sigma^2}(b_j - R_{x_{t-1}})(x_t - R_{x_{t-1}})\right)/Z_t, \quad j = 1, 2, 3 \]
Theorem. (existence and stability steady state)
Assume opposite bias, i.e. $b_2 > 0 > b_3$, then (1) has unique steady state $E$, which equals the fundamental steady state when $b_2 = -b_3$.

$E$ exhibits a Hopf bifurcation for $\beta = \beta^*$:
$E$ stable for $0 < \beta < \beta^*$ and
$E$ unstable for $\beta > \beta^*$.

Corollary: Biased beliefs lead to a different route to complexity
Three Type Example

**Theorem** (neoclassical limit, i.e. $\beta = \infty$)

When biased beliefs are exactly opposite, i.e. $b_2 = -b_3 = b > 0$, then (1) has *globally stable 4-cycle*.

For all three types, average profit along this 4-cycle is $b^2$.

**Corrolary**

Fundemantalists with zero costs and infinite memory can not beat opposite biased beliefs!
Bifurcation diagram and largest Lyapunov exponent plot

**Figure:** Bifurcation diagram (top panel) and largest Lyapunov exponent plot (bottom panel) for 3-type model with fundamentalists versus optimists and pessimists.
Figure: Phase plot for 3-type model with fundamentalists versus optimists and pessimists, as in Brock and Hommes, 1998, Figures 7 and 8.
The model
3-type model: fundamentalists vs. optimists and pessimists

Time series

Figure: Time series for 3-type model with fundamentalists versus optimists and pessimists, as in Brock and Hommes, 1998, Figures 7 and 8.
The model

4-type model: fundamentalists vs. trend and bias

Four Belief Types

(zero costs; memory one lag)

example.

\[
g_1 = 0 \quad b_1 = 0 \quad \text{fundamentalists}
\]

\[
g_2 = 0.9 \quad b_2 = 0.2 \quad \text{trend + upward bias}
\]

\[
g_3 = 0.9 \quad b_3 = -0.2 \quad \text{trend + downward bias}
\]

\[
g_4 = R = 1.01 \quad b_4 = 0 \quad \text{trend chaser}
\]

\[
R_{xt} = \sum_{h=1}^{4} n_{h,t} (g_h x_{t-1} + b_j)
\]

\[
n_{h,t+1} = \exp\left(\frac{\beta}{a\sigma^2} (g_h x_{t-2} + b_h - R_{xt-1})(x_t - R_{xt-1})) / Z_t, \quad h = 1, 2, 3, 4\right)
\]
Four Belief Types

**Rational Route to Randomness:**
- $\beta < \beta^*$: fundamental steady state globally stable
- $\beta = \beta^*$: **Hopf bifurcation** of steady state
- $\beta^* < \beta < \beta^{**}$: periodic and quasi-periodic price fluctuations on attracting invariant circle
- high values of $\beta$: strange attractors
- $\beta = \infty$: convergence to (locally unstable) fundamental steady state

**Theoretical Question:**
Is the system close to *homoclinic orbits* and chaos, when the intensity of choice $\beta$ is high?
The model

4-type model: fundamentalists vs. trend and bias

Bifurcation diagram and largest Lyapunov exponent plot

Figure: Bifurcation diagram (top panel) and largest Lyapunov exponent plot (bottom panel) for 4-type model.
The model

4-type model: fundamentalists vs. trend and bias

Chaotic and noisy chaotic time series, and strange attractor

Figure: Chaotic (top left) and noisy chaotic (top right) time series of asset prices in adaptive belief system with four trader types. Strange attractor (bottom left) and enlargement of strange attractor (bottom right).
Forecasting errors

Figure: Forecasting errors for nearest neighbor method applied to chaotic returns series (lowest graph) as well as noisy chaotic returns series, for time horizons 1 – 20 and for different noise levels, in ABS with four trader types.

Price-to-Earnings

Price-to-Dividends
S&P 500, 1950-2012 + benchmark fundamental

\[ p_t^* = \frac{1+g}{1+r} y_t \] (g constant growth rate dividends)
BH-Model and risk premium

market clearing (with zero net supply)

\[
\sum_{h=1}^{H} n_{h,t} \frac{E_{h,t}[p_{t+1} + y_{t+1}] - (1 + r)p_t}{aV_t[R_{t+1}]} = 0
\]

equilibrium pricing equation

\[
pt = \frac{1}{1 + r} \sum_{h=1}^{H} n_{h,t} E_{h,t}(p_{t+1} + y_{t+1}), \quad \text{or } r = \sum_{h=1}^{H} n_{h,t} \frac{E_{h,t}[p_{t+1} + y_{t+1} - p_t]}{p_t},
\]

estimation:
required rate of return \( r = \text{risk free interest rate} + \text{risk premium} \)
Stochastic cash flow with constant growth rate

\[ \log y_t \text{ Gaussian random walk with drift:} \]

\[ \log y_{t+1} = \mu + \log y_t + \nu_{t+1}, \quad \nu_{t+1} \sim \text{i.i.d. } N(0, \sigma^2_v), \]

This implies

\[ \frac{y_{t+1}}{y_t} = e^{\mu + \nu_{t+1}} = e^{\mu + \frac{1}{2} \sigma^2_v} e^{\nu_{t+1} - \frac{1}{2} \sigma^2_v} = (1 + g)\epsilon_{t+1}, \]

where \( g = e^{\mu + \frac{1}{2} \sigma^2_v} - 1 \) and \( \epsilon_{t+1} = e^{\nu_{t+1} - \frac{1}{2} \sigma^2_v} \), which implies \( E_t(\epsilon_{t+1}) = 1 \).

all types correct beliefs about cash flows

\[ E_{h,t}[y_{t+1}] = E_t[y_{t+1}] = (1 + g)y_t E_t[\epsilon_{t+1}] = (1 + g)y_t. \]
Empirical Validation

RE fundamental benchmark for constant growth cash flow

\[ p_t = \frac{1}{1 + r} E_t(p_{t+1} + y_{t+1}) \]

"no bubble" condition implies unique bounded RE fundamental price \( p_t^* \):
(discounted sum of expected future dividends)

\[ p_t^* = \frac{E_t(y_{t+1})}{1 + r} + \frac{E_t(y_{t+2})}{(1 + r)^2} + \cdots = \frac{1 + g}{1 + r} y_t + \frac{(1 + g)^2}{(1 + r)^2} y_t + \cdots = \frac{1 + r}{r - g} y_t. \]

fundamental price to cash flow ratio

\[ \delta_t^* = \frac{p_t^*}{y_t} = \frac{1 + r}{r - g} = m \]
Reformulation BH-model in terms of price to cash flows

**equilibrium pricing equation**

\[ p_t = \frac{1}{1 + r} \sum_{h=1}^{H} n_{h,t} E_{h,t} (p_{t+1} + y_{t+1}) \]

in terms of **price-to-cash flows**  \( \delta_t = \frac{p_t}{y_t} \)

\[ \delta_t = \frac{1}{R^*} \{ 1 + \sum_{h=1}^{H} n_{h,t} E_{h,t} [\delta_{t+1}] \}, \quad R^* = \frac{1 + r}{1 + g} \]
Heterogeneous Beliefs in terms of price-to-cash flows

deviation price-to-cash flow from fundamental

\[ x_t = \delta_t - m = \delta_t - \frac{1 + g}{r - g} \]

belief of type h about price-to-cash flow:

\[ E_{ht}[\delta_{t+1}] = E_t[\delta^*_t] + f_h(x_{t-1}, \ldots, x_{t-L}) = m + f_h(x_{t-1}, \ldots, x_{t-L}) \]

pricing equation in deviations from fundamental

\[ R^* x_t = \sum_{h=1}^{H} n_{ht} f_h(x_{t-1}, \ldots, x_{t-L}), \quad R^* = \frac{1 + r}{1 + g} \]
Evolutionary Fitness Measure

realized net profits in period $t$

$$U_{ht} = \pi_{ht} = R_t z_{h,t-1} = (p_t + y_t - R p_{t-1}) \frac{E_{h,t-1}[p_t + y_t - R p_{t-1}]}{aV_{t-1}[p_t + y_t - R p_{t-1}]}$$

Assume (in analogy with BH)

$$V_{t-1}[p_t + y_t - R p_{t-1}] = V_{t-1}[p^*_{t} + y_t - R p^*_{t-1}] = y^2_{t-1} \eta^2$$

fitness in deviations from fundamental

$$U_{ht} = \pi_{ht} = \frac{(1 + g)^2}{a \eta^2} (x_t - R^* x_{t-1}) (E_{h,t-1}[x_t - R^* x_{t-1}])$$
Two types: Fundamentalists versus trend

Two trader types, with forecasting rules

\[ f_{1t} = \phi_1 x_{t-1}, \quad 0 \leq \phi_1 < 1 \quad \text{fundamentalists} \]

\[ f_{2t} = \phi_2 x_{t-1}, \quad \phi_2 > 1, \quad \text{trend extrapolators} \]
Fractions of belief types are updated in each period according to discrete choice model (BH 1997, 1998)

\[
n_{h,t} = \frac{\exp[\beta \pi_{h,t-1}]}{\sum_{k=1}^{H} \exp[\beta \pi_{k,t-1}]} = \frac{1}{1 + \sum_{k \neq h} \exp[-\beta \Delta \pi_{h,k}^{t-1}]},
\]

where \( \beta > 0 \) is intensity of choice and \( \Delta \pi_{h,k}^{t-1} = \pi_{h,t-1} - \pi_{k,t-1} \) difference in realized profits types \( h \) and \( k \).

In 2-type case, fraction of type 1:

\[
n_{t} = \frac{1}{1 + \exp \{-\beta^* [((\phi_1 - \phi_2)x_{t-3}x_{t-1} - R^*x_{t-2})]\}}
\]
Estimation of two type model in deviations from fundamental; synchronous updating; Boswijk et al., JEDC 2007

\[ R^* x_t = n_t \phi_1 x_{t-1} + (1 - n_t) \phi_2 x_{t-1} + \epsilon_t \]

\[ R^* = \frac{1 + r}{1 + g} \approx 1.074 \]

- \( \phi_1 = 0.762 \): fundamentalists, mean reversion
- \( \phi_2 = 1.135 \) trend extrapolators
- \( \beta \approx 10 \)

\[ \phi_t = \frac{n_t \phi_1 + (1 - n_t) \phi_2}{R^*} \]

market sentiment

- \( \phi_t < 1 \): mean reversion;
- \( \phi_t > 1 \): explosive, trend following
Fraction Fundamentalists & Market Sentiment

Explanation: dot com bubble triggered by economic fundamentals and strongly amplified by trend following behavior
Empirical Validation

Average Response to Fundamental shock (2000 runs)

short term overreaction and long term mean reversion
Quantiles of 2000 simulated predictions of the PE-ratio in deviations from fundamental
Financial Crisis not Extreme in Nonlinear Switching Model

Quantiles of 2000 simulated predictions of the PE-ratio in deviations from fundamental
Asset Pricing Model with Heterogeneous Beliefs

- rational route to randomness as $\beta$ increases, with temporary bubbles and crashes
  - weak correlation of beliefs: stable price behavior;
  - strong coordination of beliefs: unstable price dynamics
- counter-examples to Friedman hypothesis: fundamentalists do not drive out “irrational” technical analysts, driven by short run profits
- empirical validation: explanation of bubbles and crashes
- consistent with learning-to-forecast laboratory experiments